

FIXED POINTS FOR φ -CONTRACTIONS IN E -BANACH SPACES

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Abstract. In this work we will present some extensions of Banach-Caccioppoli fixed point principle for classes of φ -contractions in E -metric spaces. Also, we extend Krasnoselskii's fixed point theorem to the case of E -Banach spaces and we give an application to a Fredholm-Volterra type differential equation where one of the integral operators satisfies a φ -contraction condition.

Key Words and Phrases: Contraction principle, E -Banach space, E -metric space, fixed point, Fredholm-Volterra equation, integral equation, Krasnoselskii's theorem, φ -contraction, Picard operator, Riesz space, Schauder's theorem, sum of two operators, vector Banach space, vector metric space.

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