POSITIVE-ADDITIVE FUNCTIONAL EQUATIONS IN C^*-ALGEBRAS

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Abstract. In this paper, we introduce a positive-additive functional equation in C^*-algebras. Using fixed point methods, we prove the stability of the positive-additive functional equation in C^*-algebras. Moreover, we prove the Hyers-Ulam stability of the positive-additive functional equation in C^*-algebras by the direct method of Hyers and Ulam.
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