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AN EXISTENCE-UNIQUENESS THEOREM FOR A CLASS OF BOUNDARY VALUE PROBLEMS

M.R. MOKHTARZADEH*, M.R. POURNAKI**,1 AND A. RAZANI***,2

*School of Mathematics, Institute for Research in Fundamental Sciences (IPM)
P.O. Box 19395-5746, Tehran, Iran
E-mail: mrmokhtarzadeh@ipm.ir

**Department of Mathematical Sciences, Sharif University of Technology
P.O. Box 11155-9415, Tehran, Iran
E-mail: pournaki@ipm.ir

***Department of Mathematics, Faculty of Science, Imam Khomeini International University, P.O. Box 34194-288, Qazvin, Iran, and
School of Mathematics, Institute for Research in Fundamental Sciences (IPM)
P.O. Box 19395-5746, Tehran, Iran
E-mail: razani@ikiu.ac.ir

Abstract. In this paper the solutions of a two–endpoint boundary value problem is studied and under suitable assumptions the existence and uniqueness of a solution is proved. As a consequence, a condition to guarantee the existence of at least one periodic solution for a class of Liénard equations is presented.
Key Words and Phrases: Nonlinear boundary value problem, Liénard equation, periodic solution, Banach space, Schauder's fixed point theorem.
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