

THE MAJORIZATION FIXED POINT PRINCIPLE AND APPLICATIONS TO NONLINEAR INTEGRAL EQUATIONS

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Abstract. In this article we consider a modification of the Kantorovich and Banach – Caccioppoli fixed point principles for operators satisfying a local Lipschitz condition. In particular, we give sharp estimates for the inner and outer radius of existence and uniqueness. Furthermore, new *a priori* and *a posteriori* error estimates for successive approximations are deduced. Our examples reflect the fact that many existence and uniqueness results for fixed points of operators in the spaces C and L_p can be strengthened by passing to local Lipschitz conditions.

Key Words and Phrases: Banach – Caccioppoli fixed point principle, Kantorovich majorant fixed point principle, local Lipschitz condition, *a priori* and *a posteriori* error estimates, equations with polynomial operators, mixed Hammerstein equations, generalized Uryson equations.

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