ON THE EXISTENCE OF POSITIVE SOLUTIONS
OF A NONLINEAR q-DIFFERENCE EQUATION

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Abstract. This paper is concerned with a boundary value problem of the nonlinear q-difference
equation $-D_q^2 u(t) = f(t, u(t))$, with some boundary conditions. Under certain conditions on f, the
existence of positive solutions is obtained by applying a fixed point theorem in cones.
Key Words and Phrases: Boundary-value problem, q-difference equation, Green's function, Krasnoselskii's fixed-point theorem.
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