POSITIVE SOLUTIONS FOR SECOND ORDER DIFFERENTIAL SYSTEMS WITH NONLOCAL CONDITIONS

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Abstract. A class of second order differential systems with nonlocal conditions

$$\begin{cases} u''(t) + f_1(t, u(t), v(t)) = 0, & t \in (0, 1), \\ v''(t) + f_2(t, u(t), v(t)) = 0, & t \in (0, 1), \\ u'(0) = v'(0) = 0, \ u(1) = \alpha u(\eta), \ v(1) = \alpha v(\eta) \end{cases}$$

is considered under some conditions concerning the first eigenvalue of the relevant linear problem. By constructing a cone $K_1 \times K_2$ which is the Cartesian product of two cones and computing the fixed point index in $K_1 \times K_2$, the existence of positive solutions for the systems is established. An example is provided to illustrate the main results.

Key Words and Phrases: Positive solution, nonlocal boundary value problem, differential systems, cone, fixed point index.

2010 Mathematics Subject Classification: 34B10, 34B18, 47H10.

REFERENCES

[1] C. Bai, *Triple positive solutions of three-point boundary value problems for fourth-order differential equations*, Comput. Math. Appl., **56**(2008), no. 5, 1364-1371.

[2] A. Canada, A. Zertiti, *Fixed point theorems for systems of equations in ordered Banach spaces with applications to differential and integral equations*, Nonlinear Anal., **27**(1996), no. 4, 397-411.

[3] X. Cheng, *Existence of positive solutions for a class of second-order ordinary differential systems*, Nonlinear Anal., **69**(2008), no. 9, 3042-3049.

[4] X. Cheng, C. Zhong, *Existence of positive solutions for a second-order ordinary differential system*, J. Math. Anal. Appl., **312**(2005), no. 1, 14-23.

[5] G.R. Gavalas, *Nonlinear Differential Equations of Chemically Reacting Systems*, Spinger-Verlag, New York, 1968.

[6] D. Guo, V. Lakshmikantham, *Nonlinear Problems in Abstract Cones*, Academic Press, New York, 1988.

[7] J. Henderson, S.K. Ntouyas, I.K. Purnaras, *Positive solutions for systems of m-point nonlinear boundary value problems*, Math. Modelling Anal., **13**(2008), no. 3, 357-370.

[8] A.W. Leung, *Systems of Nonlinear Partial Differential Equations*, Kluwer Academic Publishers, Dordrecht, 1989.

[9] B. Liu, *Positive solutions of a nonlinear three-point boundary value problem*, Appl. Math. Comput., **132**(2002), no. 1, 11-28.

[10] B. Liu, L. Liu, Y. Wu, *Positive solutions for singular systems of three-point boundary value problems*, Comput. Math. Appl., **53**(2007), no. 9, 1429-1438.

[11] R. Ma, H. Wang, *Positive solutions of nonlinear three-point boundary value problems*, J. Math. Anal. Appl., **279**(2003), no. 1, 216-227.

[12] R. Precup, *A vector version of Krasnosel'skii's fixed point theorem in cones and positive periodic solutions of nonlinear systems*, J. Fixed Point Theory Appl., **2**(2007), no. 1, 141-151.

[13] Y. Sun, *Existence and multiplicity of symmetric positive solutions for three-point boundary value problem*, J. Math. Anal. Appl., **329**(2007), no. 2, 998-1009.

[14] Y. Sun, *Existence of triple positive solutions for a third-order three-point boundary value problem*, J. Comput. Appl. Math., **221**(2008), no. 1, 194-201.

[15] Y. Sun, *Optimal existence criteria for symmetric positive solutions to a three-point boundary value problem*, Nonlinear Anal., **66**(2007), no. 5, 1051-1063.

[16] Y. Sun, *Positive solutions for third-order three-point nonhomogeneous boundary value problems*, Appl. Math. Lett., **22**(2009), no. 1, 45-51.

[17] S. Wang, J. Liu, *Coexistence of of positive solutions of nonlinear three-point boundary value and its conjugate problem*, J. Math. Anal. Appl., **330**(2007), no. 1, 334-351.

[18] J.R.L. Webb, *Positive solutions of some three-point boundary value problems via fixed point index theory*, Nonlinear Anal., **47**(2001), no. 7, 4319-4332.

[19] J. Wei, J. Sun, *Positive solutions to systems of nonlinear second-order three-point boundary value problems*, Appl. Math. E-Notes, **9**(2009), no. 1, 55-62.

[20] Q. Yao, *Successive iteration and positive solution for nonlinear second-order three-point boundary value problems*, Comput. Math. Appl., **50**(2005), no. 3-4, 433-444.

[21] J. Zhao, W. Ge, *A necessary and sufficient condition for the existence of positive solutions to a kind of singular three-point boundary value problem*, Nonlinear Anal., **71**(2009), no. 9, 3973-3980.

[22] Y. Zhou, Y. Xu, *Positive solutions of three-point boundary value problems for systems of nonlinear second order ordinary differential equations*, J. Math. Anal. Appl., **320**(2006), no. 2, 578-590.