Fixed Point Theory, 13(2012), No. 2, 461-474
http://www.math.ubbcluj.ro/~nodeacj/sfptcj.html

FURI-PERA FIXED POINT THEOREMS FOR NONEXPANSIVE MAPS IN BANACH SPACES

SMAÏL DJEBALI* AND KARIMA HAMMACHE**

*Department of Mathematics, École Normale Supérieure
Po. Box 92, 16050 Kouba, Algiers, Algeria
E-mail: djebali@ens-kouba.dz

**Department of Mathematics, École Normale Supérieure
Po. Box 92, 16050 Kouba, Algiers, Algeria
E-mail: k.hammache@hotmail.com

Abstract. In this work, we present some new fixed point theorems for nonexpansive maps, 1-set contractions, and demi-closed nonexpansive perturbations of nonexpansive maps defined on closed, convex, not necessarily bounded subsets of Banach spaces; the stress will be made on the so-called Furi-Pera boundary condition. The proofs use the Kuratowski measure of noncompactness and rely on a recent compactness result for the approximate fixed point set. To illustrate the results obtained, applications to a fixed point theorem in a Banach algebra and to an integral equation are provided.
Key Words and Phrases: Nonexpansive map, 1-set contraction, Furi-Pera condition, demi-closed map, Kuratowski MNC, fixed point.
2010 Mathematics Subject Classification: 47H09, 34B18, 47H10, 47J25.

References

[1] R.P. Agarwal, M. Meehan, D. O'Regan, *Fixed Point Theory and Applications,* Cambridge University Press, 2001.
[2] R.P. Agarwal, D. O'Regan, D.R. Sahu, *Fixed Point Theory for Lipschitzian-type Mappings with Applications,* Springer, New York, 2009.
[3] J. Banas, K. Goebel, *Measure of Noncompactness in Banach Spaces,* Marcel Dekker, New York, 1980.
[4] F.E. Browder, *Semicontractive and semiaccretive nonlinear mappings in Banach spaces,* Bull. Amer. Math. Soc., **74**(1968), 660–665.
[5] R.E. Bruck, *Properties of fixed-point sets of nonexpansive mappings in Banach spaces,* Trans. Amer. Math. Soc., **179**(1973), 251–262.
[6] R.E. Bruck, *A common fixed point theorem for a commuting family of nonexpansive mappings,* Pacific J. Math., **53**(1974), 59–71.
[7] C. Chidume, *Geometric Properties of Banach Spaces and Nonlinear Iterations,* Springer Verlag, 2009.
[8] I. Ciorănescu, *Geometry of Banach Spaces, Duality Mappings and Nonlinear Problems,* Kluwer Academic, 1990.
[9] G. Darbo, *Punti uniti in transformationi a condominio non-compactto,* Rend. Sem. Mat. Univ. Padova, **24**(1955), 84–92.
[10] K. Deimling, *Nonlinear Functional Analysis,* Springer Verlag, 1985.

[11] B.C. Dhage, *On a fixed point theorem in Banach algebras with applications,* Appl. Math. Lett., **18**(2005) 273–280.
[12] S. Djebali, K. Hammache, *Furi-Pera fixed point theorems in Banach algebras with applications,* Acta. Univ. Palacki. Olomuc, Fac. Rer. Nat., Mathematica **47**(2008), 55–75.
[13] S. Djebali, K. Hammache, *Fixed point theorems for nonexpansive maps in Banach spaces,* Nonlinear Anal., **73**(2010), 3440–3449.
[14] J. Dugundji, A. Granas, *Fixed Point Theory,* Springer Monographs in Mathematics, Springer Verlag, New York, 2003.
[15] M. Edelstein, *On nonexpansive mappings,* Proc. Amer. Math. Soc., **15**(1984), 689–695.
[16] M. Furi, P. Pera, *A continuation method on locally convex spaces and applications to ODE on noncompact intervals,* Annales Polon. Math., **XLVII**(1987), 331–346.
[17] K. Goebel, *An elementary proof of the fixed point theorem of Browder and Kirk,* Michigan Math. J., **16**(1969), 381–383.
[18] K. Goebel, W.A. Kirk, *Topics in Metric Fixed Point Theory,* Cambridge Studies in Adv. Math., **28**, Cambridge University Press, 1990.
[19] K. Goebel, W.A. Kirk, *Some problems in metric fixed point theory,* J. Fixed Point Theory Appl., **4**(2008), 13–25.
[20] K. Goebel, S. Reich, *Uniform Convexity, Hyperbolic Geometry, and Nonexpansive Mappings,* Monographs and Textbooks in Pure and Applied Mathematics, **83**, Marcel Dekker, New york, 1984.
[21] D. Göhde, *Zum prinzip der kontraktiven abbildung,* Math. Nachr., **30**(1965), 251–258.
[22] G. Isac, S.Z. Nemeth, *A fixed point theorem for asymptotically contractive mapping,* J. Math. Anal. Appl., **314**(2005), 500–512.
[23] G. Isac, S.Z. Nemeth, *Fixed points and positive eigenvalues for nonlinear operators,* J. Math. Anal. Appl., **314**(2006), 500–512.
[24] V.I. Istrăţescu, *Fixed Point Theory. An Introduction*, D. Reidel Publishing Company, Dordrecht, Boston, London, 1979.
[25] H. Jeggle, *Nichtlineare Funktional Analysis,* Teubener Stuttgart, 1979.
[26] A. Kaewcharoen, W.A. Kirk, *Nonexpansive mapping defined on unbounded domains,* Fixed Point Theory Appl., **2006**(2006), Article ID 82080, 1–13.
[27] W.A. Kirk, *A fixed point theorem for mappings which do not increase distance,* Amer. Math. Monthly, **72**(1965), 1002–1004.
[28] W.A. Kirk, *An abstract fixed point theorem for nonexpansive mappings,* Proc. Amer. Math. Soc., **82**(1981), 640–642.
[29] D. O'Regan, *Fixed-point theory for the sum of two operators,* Appl. Math Lett., **9(1)**(1996), 1–8.
[30] J.P. Penot, *A fixed-point theorem for asymptotically contractive mappings,* Proc. Amer. Math. Soc., **131(8)**(2003), 2371–2377.
[31] W.O. Ray, *The fixed point property and unbounded sets in Hilbert spaces,* Trans. Amer. Math. Soc., **131(8)**(2003), 2371–2377.
[32] D.R Smart, *Fixed Point Theorems,* Cambridge University Press, 1974.
[33] R. Sine, *On the converse of the nonexpansive map fixed point theorem for Hilbert spaces,* Proc. Amer. Math. Soc., **100(3)**(1987), 489–490.
[34] K. Yoshida, *Functional Analysis, third edition*, Springer Verlag, Berlin, 1971.
[35] E. Zeidler, *Nonlinear Functional Analysis and its Applications. Vol. I: Fixed Point Theorems,* Springer-Verlag, New York, 1986.

Received: January 4, 2011; Accepted: March 9, 2011.