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EXISTENCE OF POSITIVE SOLUTIONS OF BOUNDARY VALUE PROBLEMS FOR SECOND-ORDER FUNCTIONAL DIFFERENTIAL EQUATIONS ON INFINITE INTERVALS

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Abstract. In present paper, the author investigates the existence of positive solutions of boundary value problems for second-order functional differential equations on infinite intervals as follows

$$\begin{cases} x'' - p(t)x' - q(t)x + f(t, x_t, x'_t) = 0, t \in I = [0, \infty), \\ \alpha x(t) - \beta x'(t) = \xi(t) \geq 0, t \in [-\tau, 0], \xi(0) = x(\infty) = 0, \end{cases}$$

where $\alpha \geq 0, \beta > 0, \xi(t) \in C[-\tau, 0]$. By applying fixed point index theorem on cone and operator spectra theorem, the author obtains the results on existence of positive solutions of boundary value problems.

Key Words and Phrases: Functional differential equation, positive solution, fixed point index on cone, operator spectra theorem.
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