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MANN TYPE HYBRID EXTRAGRADIENT METHOD FOR VARIATIONAL INEQUALITIES, VARIATIONAL INCLUSIONS AND FIXED POINT PROBLEMS

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Abstract. Recently, Nadezhkina and Takahashi [N. Nadezhkina, W. Takahashi, Strong convergence theorem by a hybrid method for nonexpansive mappings and Lipschitz-continuous monotone mappings, SIAM J. Optim. 16 (4) (2006) 1230-1241] introduced an iterative algorithm for finding a common element of the fixed point set of a nonexpansive mapping and the solution set of a variational inequality in a real Hilbert space via combining two well-known methods: hybrid and extragradient. In this paper, we investigate the problem of finding a common solution of a variational inequality, a variational inclusion and a fixed point problem of a nonexpansive mapping in a real Hilbert space. Motivated by Nadezhkina and Takahashi's hybrid-extragradient method we propose and analyze Mann type hybrid-extragradient algorithm for finding a common solution. It is proven that three sequences generated by this algorithm converge strongly to the same common solution under very mild conditions. Based on this result, we also construct an iterative algorithm for finding a common fixed point of three mappings, such that one of these mappings is nonexpansive and the other two mappings are taken from the more general class of Lipschitz pseudocontractive mappings and from the more general class of strictly pseudocontractive mappings, respectively.

Key Words and Phrases: Variational inclusion, variational inequality, fixed point, nonexpansive mapping, inverse strongly monotone mapping, maximal monotone mapping, strong convergence.
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