STRONG CONVERGENCE THEOREMS FOR GENERAL VARIATIONAL INEQUALITY PROBLEMS AND FIXED POINT PROBLEMS IN q-UNIFORMLY SMOOTH BANACH SPACES

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Abstract. In this paper, we introduce a new iterative algorithm for finding a common element of the set of solutions of a general variational inequality and the set of common fixed points of an infinite family of nonexpansive mappings in q-uniformly smooth Banach space. We obtain some strong convergence theorems under suitable conditions. Furthermore we give an appropriate example such that all conditions of this result are satisfied. Our results extend the recent results announced by many others.
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