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TRUNCATION METHOD FOR INFINITE COUNTABLE SYSTEMS OF PARABOLIC DIFFERENTIAL-FUNCTIONAL EQUATIONS

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Abstract. In the paper we study the truncation method for solvability of the Fourier first boundary problem for infinite countable systems of nonlinear parabolic-reaction-diffusion equations with Volterra functionals in Banach sequences spaces. These systems arise as discrete models of processes considered. In the truncation method a solution of the infinite countable system is defined as the limit when $N \to \infty$ of the sequence of approximations $\{z_N\}_{N=1,2,\ldots}$, where $z_N = (z_N^1, z_N^2, \ldots, z_N^N)$ are defined as solutions of the finite systems of the first N equations in N unknown functions with corresponding initial and boundary conditions. The truncation method plays an important role among approximation methods; it is very useful and commonly used in numerical computation of approximate solutions. The main results of the paper are an existence and uniqueness theorem for infinite countable systems of nonlinear parabolic-reaction-diffusion equations with Volterra functionals and a new method for the construction of truncated systems when a lower or an upper solution of the problem considered is known. This method may be used also to research of positive solutions of discrete models in infinite-dimensional Banach spaces.
Key Words and Phrases: Truncation method, truncated system, infinite countable system, infinite uncountable system, differential-functional equation, reaction-diffusion equation, uniformly parabolic equation, Banach sequence space, partially ordered Banach space, Volterra functional, positive solution.
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