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A NOTE ON STABILITY OF THE LINEAR FUNCTIONAL EQUATIONS OF HIGHER ORDER AND FIXED POINTS OF AN OPERATOR

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Abstract. We prove two general theorems, which appear to be very useful in investigation of the Hyers-Ulam stability of a higher order linear functional equation in single variable, with constant coefficients. We give several examples of their applications. In particular we show that we obtain in this way several fixed point results for a particular operator. The main tool in the proofs is a complexification of a real normed (or Banach) space X, which can be described as the tensor product $X \otimes \mathbb{R}^2$ endowed with the Taylor norm.
Key Words and Phrases: Hyers-Ulam stability, nonstability, fixed point, linear equation, characteristic equation, complexification, Banach space.
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