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ON ROLEWICZ-ZABCZYK TECHNIQUES IN THE STABILITY THEORY OF DYNAMICAL SYSTEMS

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Dedicated to Professor Ioan A. Rus on the occasion of his 75th birthday

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Abstract. The aim of this paper is to present a general overview concerning the Rolewicz-Zabczyk type techniques in the stability theory of dynamical systems. We discuss the main methods based on trajectories that may be used in order to characterize the uniform exponential stability of variational discrete systems and their applications to the case of skew-product flows. Beside our techniques used in the past decade on this topic, we also point out several new issues and analyze both their connections with previous results as well as some new characterizations for uniform exponential stability. Finally, motivated by the potential extension of the framework to dichotomy, we propose several open problems in the case of the exponential instability.

Key Words and Phrases: variational difference equation; exponential stability; skew-product flow; translation invariant sequence space.
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