

## CONTRACTION MAPPINGS IN FUZZY QUASI-METRIC SPACES AND $[0,1]$ -FUZZY POSETS

PEDRO TIRADO

Instituto Universitario de Matemática Pura y Aplicada  
Universidad Politécnica de Valencia  
46071 Valencia, Spain  
E-mail: pedtipe@mat.upv.es

**Abstract.** It is well known that each bounded ultraquasi-metric on a set induces, in a natural way, an  $[0,1]$ -fuzzy poset. On the other hand, each  $[0,1]$ -fuzzy poset can be seen as a stationary fuzzy ultraquasi-metric space for the continuous t-norm Min. By extending this construction to any continuous t-norm, a stationary fuzzy quasi-metric space is obtained. Motivated by these facts, we present several contraction principles on fuzzy quasi-metric spaces that are applied to the class of spaces described above. Some illustrative examples are also given. Finally, we use our approach to deduce in an easy fashion the existence and uniqueness of solution for the recurrence equations typically associated to the analysis of Probabilistic Divide and Conquer Algorithms.

**Key Words and Phrases:** fuzzy quasi-metric,  $[0,1]$ -fuzzy poset, contraction mapping, fixed point, recursive equation.

**2010 Mathematics Subject Classification:** 47H10, 54A40, 54H25, 68Q25, 68Q55.

**Acknowledgments.** The author thanks the support of the Spanish Ministry of Science and Innovation, grand MTM2009-12872-C02-01. The author also thanks the referees because their suggestions and remarks have allowed to improve the first version of this paper.

### REFERENCES

- [1] R. Engelking, *General Topology*, Polish Sci. Publ., Warszawa, 1977.
- [2] L. Fan, *A new approach to quantitative domain theory*, *Electronic Notes Theoret.* **45** (2001), URL: <http://www.elsevier.nl/locate/entcs/volume45.html> 11 pages.
- [3] P. Flajolet, *Analytic analysis of algorithms*, in: 19th Internat. Colloq. ICALP'92, Vienna, July 1992; Automata, Languages and Programming, Lecture Notes in Computer Science 623, W. Kuich editor (1992), pp. 186–210.
- [4] A. George, P. Veeramani, *On some results in fuzzy metric spaces*, *Fuzzy Sets and Systems*, **64**(1994), 395-399.
- [5] A. George, P. Veeramani, *On some results of analysis of fuzzy metric spaces*, *Fuzzy Sets and Systems*, **90**(1997), 365-368.
- [6] G. Gierz, K.H. Hofmann, K. Keimel, J.D. Lawson, M.W. Mislove, D.S. Scott, *Continuous Lattices and Domains*, *Encyclopedia of Mathematics and its Applications*, vol. 93, Cambridge Univ. Press, 2003.
- [7] M. Grabiec, *Fixed points in fuzzy metric spaces*, *Fuzzy Sets and Systems*, **27**(1988), 385-389.
- [8] V. Gregori, A. López-Crevillente, S. Morillas, A. Sapena, *On convergence in fuzzy metric spaces*, *Topol. Appl.*, **156**(2009) 3002-3006.

- [9] V. Gregori, S. Morillas, A. Sapena, *On a class of completable fuzzy metric spaces*, Fuzzy Sets and Systems, **161**(2010), 2193-2205.
- [10] V. Gregori, S. Romaguera, *Fuzzy quasi-metric spaces*, Appl. Gen. Topology, **5**(2004), 129-136.
- [11] V. Gregori, A. Sapena, *On fixed point theorems in fuzzy metric spaces*, Fuzzy Sets and Systems, **125**(2002), 245-253.
- [12] P. Hájek, *Metamathematics of Fuzzy Logic*, Kluwer, Dordrecht, 1998.
- [13] P. Hájek, *Basic fuzzy and BL-algebras*, Soft Computing, **2**(1998), 124-128.
- [14] I. Kramosil, J. Michalek, *Fuzzy metrics and statistical metric spaces*, Kybernetika, **11**(1975), 336-344.
- [15] H.P.A. Künzi, *Nonsymmetric distances and their associated topologies: About the origin of basic ideas in the area of asymmetric topology*, in: Handbook of the History of General Topology, Volume 3 (C. E. Aull and R. Lowen eds.), Kluwer, Dordrecht (2001), 853-968.
- [16] D. Mihet, *A Banach contraction theorem in fuzzy metric spaces*, Fuzzy Sets and Systems, **144**(2004), 431-439.
- [17] D. Mihet, *Fuzzy  $\psi$ -contractive mappings in non-Archimedean fuzzy metric spaces*, Fuzzy Sets and Systems, **159**(2008), 739-744.
- [18] D. Mihet, *A class of contractions in fuzzy metric spaces*, Fuzzy Sets and Systems, **161**(2010), 1131-1137.
- [19] S. Romaguera, A. Sapena, P. Tirado, *The Banach fixed point theorem in fuzzy quasi-metric spaces with application to the domain of words*, Topology Appl., **154**(2007), 2196-2203.
- [20] S. Romaguera, P. Tirado, *On fixed point theorems in intuitionistic fuzzy metric spaces*, Internat. J. Nonlinear Sci. Numer. Simul., **8**(2007), 233-238.
- [21] A. Sapena, *A contribution to the study of fuzzy metric spaces*, Appl. Gen. Topology, **2**(2001), 63-76.
- [22] B. Schweizer, A. Sklar, *Statistical metric spaces*, Pacific J. Math., **10**(1960), 314-334.
- [23] H. Sherwood, *Complete Probabilistic Metric Spaces and Random Variables Generated Spaces*, Ph.D. Thesis, University of Arizona, 1965.
- [24] P. Tirado, *Fixed point theorems in stationary fuzzy quasi-metric spaces and  $[0,1]$ -fuzzy posets*, in: VII Iberoamerican Conf. on Topology and its Applications, Valencia, Spain, 25-28 June 2008, Book of Abstracts, pp. 37-38.
- [25] R. Vasuki, P. Veeramani, *Fixed point theorems and Cauchy sequences in fuzzy metric spaces*, Fuzzy Sets and Systems, **135**(2003), 415-417.
- [26] Q. Zhang, L. Fan, *Continuity in quantitative domains*, Fuzzy Sets and Systems, **154**(2005), 118-131.

*Received: November 17, 2010; Accepted: June 29, 2011.*

