

STRONG CONVERGENCE THEOREMS BY GENERALIZED CQ METHOD IN HILBERT SPACES

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Abstract. Recently, CQ method has been investigated extensively. However, it is mainly applied to modify Mann, Ishikawa and Halpern iterations to get strong convergence. In this paper, we study the properties of CQ method and proposed a framework. Based on that, we obtain a series of strong convergence theorems. Some of them are the extensions of previous results. On the other hand, CQ method, monotone Q method, monotone C method and monotone CQ method, used to be given separately, have the following relations: CQ method TRUE \Rightarrow monotone Q method TRUE \Rightarrow monotone C method TRUE \Leftrightarrow monotone CQ method TRUE.

Key Words and Phrases: Generalized CQ method, Strong convergence, Mann's iteration process, Ishikawa's iteration process, Halpern's iteration process.

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