Fixed Point Theory, 12(2011), No. 1, 71-82
http://www.math.ubbcluj.ro/~nodeacj/sfptcj.html

FIXED POINT METHODS FOR THE STABILITY OF GENERAL QUADRATIC FUNCTIONAL EQUATION

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Abstract. In this paper we obtain the general solution and prove the stability in Banach spaces and also the stability using the alternative fixed point of quadratic functional equation:

$$f(ax+by+2cz)+f(ax+by-2cz)+f(ax-by+2cz)+f(ax-by-2cz)$$
$$=4a^2f(x)+4b^2f(y)+16c^2f(z)$$

for any fixed integers a,b,c with $a,b,c\neq 0,\pm 1$ and $a\pm b\neq 0$.
Key Words and Phrases: Stability, quadratic functional equation, Banach spaces, fixed point method.
2010 Mathematics Subject Classification: 39B82, 39B52, 47H10.

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Received: April 2, 2009; Accepted: June 6, 2010