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ON EKELAND'S VARIATIONAL PRINCIPLE IN b-METRIC SPACES

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Abstract. In this paper we prove a version of Ekeland's variational principle in b-metric spaces and, as a consequence, we obtain a Caristi type fixed point theorem.
Key Words and Phrases: Variational principle, fixed point, Caristi type theorem, b-metric space.
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