

## EXISTENCE, MULTIPLICITY OF POSITIVE SOLUTIONS FOR FOUR-POINT BOUNDARY VALUE PROBLEM WITH DEPENDENCE ON THE FIRST ORDER DERIVATIVE

LIU YANG\*, CHUNFANG SHEN AND YAN LIANG

Department of Mathematics  
Hefei Teacher's College, Hefei, 230061, People's Republic of China  
\*Email: xjiangfeng@163.com

**Abstract.** The paper consider positive solutions for second-order four-point boundary value problem

$$\begin{aligned}u''(t) + f(t, u, u') &= 0, t \in (0, 1) \\ u(0) &= \alpha u(\eta), u(1) = \beta u(\xi)\end{aligned}$$

where the first order derivative is involved in the nonlinear term explicitly. By using Krasnoselskii fixed point theorem and triple fixed point theorem, we show the existence, multiplicity of positive solutions for the problem. Some examples are given to illustrate the main results of the article.

**Key Words and Phrases:** Four point boundary value problem, positive solution, cone, fixed point.  
**2000 Mathematics Subject Classification:** 34B10, 34B15, 47H10.

### REFERENCES

- [1] R.Y. Ma, *Positive solutions for a nonlinear three-point boundary value problem*, Electronic J. Diff. Equations, **34**(1999), 1-8.
- [2] R.Y. Ma, *Positive solutions of a nonlinear  $m$ -point boundary value problem*, Comput. Math. Appl., **42**(2001), 755-765.
- [3] R.Y. Ma and N. Cataneda, *Existence of solution for nonlinear  $m$ -point boundary value problem*, J. Math. Anal. Appl., **256**(2001), 556-567.
- [4] P.K. Palamides, *Positive and monotone solutions of an  $m$ -point boundary-value problem*, Electronic Journal of Differential Equations, **18**(2002), 1-16.
- [5] X.M. He and W.G. Ge, *Triple solutions for second-order three-point boundary value problems*, J. Math. Anal. Appl., **268**(2002), 256-265.
- [6] Y.P. Guo and W.G. Ge, *Positive solutions for three-point boundary-value problems with dependence on the first order derivative*, J. Math. Anal. Appl., **290**(2004), 291-301.
- [7] D.J. Guo, *Nonlinear Functional Analysis*, Shangdong Sci. and Tech. Press, Jinan, 1985 (in Chinese).
- [8] B. Liu, *Positive solutions of a nonlinear four-point boundary value problems*, Appl. Math. Comput., **155**(2004), 179-203.
- [9] Z.Y. Wang and J.H. Zhang, *Positive solutions for one-dimensional  $p$ -Laplacian boundary value problems with dependence on the first order derivative*, J. Math. Anal. Appl., **314**(2006), 618-630.
- [10] R.W. Leggett and L.R. Williams, *Multiple positive fixed points of nonlinear operators on ordered Banach spaces*, Indiana Univ. Math. J., **28**(1979), 673-688.

- [11] X.M. He and W.G. Ge, *Triple solutions for second-order three-point boundary value problems*, J. Math. Anal. Appl., **268**(2002), 256-265.
- [12] Z.B. Bai and W.G. Ge, *Existence of three positive solutions for some second-order boundary value problems*, Comput. Math. Appl., **48**(2004), 699-707.
- [13] Z.B. Bai, W.G. Ge and Y.F. Wang, *Multiplicity results for some second-order four-point boundary-value problems*, Nonlinear Anal., **60**(2004), 491-500.
- [14] B. Liu, *Positive solutions of singular three-point boundary value problems for the one-dimensional  $p$ -Laplacian*, Comput. Math. Appl., **48**(2004), 913-925.

*Received: 28.12.2008; Accepted: 11.02.2009.*

