

SOME EXISTENCE RESULTS FOR FRACTIONAL FUNCTIONAL DIFFERENTIAL EQUATIONS

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Abstract. We consider a semilinear functional differential equation of a fractional order in a Banach space assuming that its linear part is the generator of a noncompact semigroup. It is assumed that the nonlinearity satisfies a regularity condition expressed in terms of the measures of noncompactness. The theory of condensing maps is used to obtain local and global existence results. The same approach is applied to a neutral functional differential equation.

Key Words and Phrases: Fractional derivative, fractional differential equation, functional differential equation, neutral functional differential equation, mild solution, Cauchy problem, existence theorem, measure of noncompactness, fixed point, condensing map.

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