

## NON-ARCHIMEDEAN STABILITY OF QUADRATIC EQUATIONS

ALIREZA KAMEL MIRMOSTAFEE

Department of Pure Mathematics  
Centre of Excellence in Analysis on Algebraic Structures  
Ferdowsi University of Mashhad  
P.O. Box 1159, Mashhad, Iran  
E-mail: mirmostafaei@ferdowsi.um.ac.ir

**Abstract.** We use the fixed point method to study the Hyers-Ulam-Rassias stability of the quadratic equation in non-Archimedean normed spaces. Some applications of our result will be illustrated. We will also give an example to show that some results in stability of quadratic mappings in real normed spaces are not valid in non-Archimedean normed spaces.

**Key Words and Phrases:** Hyers-Ulam-Rassias stability, quadratic equation, fixed point alternative.

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