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THE THEORY OF A METRICAL FIXED POINT THEOREM: THEORETICAL AND APPLICATIVE RELEVANCES

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Abstract. In this paper we present some examples of a theory of a metrical fixed point theorem. These theories are in connection with:

- Picard operators and weakly Picard operators
- Bessaga and Janos operators
- data dependence of the fixed points
- sequences of operators and fixed points
- operatorial inequalities
- comparison principle
- fibre generalized contractions
- well-posedness of the fixed point problem
- limit shadowing property
- applications.

Key Words and Phrases: generalized metric spaces, generalized contractions, infinite matrices, Picard operators, weakly Picard operators, Bessaga operators, Janos operators, data dependence, operatorial inequalities, comparison principle, fibre generalized contractions, topological isomorphisms, well-posedness of fixed point problem, limit shadowing property.
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