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A MODIFIED CQ METHOD FOR EQUILIBRIUM PROBLEMS, FIXED POINTS AND VARIATIONAL INEQUALITY

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Abstract. In this paper, we introduce a modified CQ iterative scheme for finding a common element of the set of solutions of an equilibrium problem, the set of fixed points of an infinite family of nonexpansive mappings and the set of the variational inequality for an α-inverse strongly monotone mapping in a Hilbert space. We obtain a strong convergence theorem for three sequences generated by this process. Based on this result, we also get several new and interesting results which generalize and extend some well-known strong convergence theorems in the literature.

Key Words and Phrases: equilibrium problem, CQ method, nonexpansive mapping, variational inequality, strong convergence, fixed point.
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