

**LERAY-SCHAUDER, LEFSCHETZ AND
KRASNOSELSKII FIXED POINT THEORY IN FRÉCHET
SPACES FOR GENERAL CLASSES OF VOLTERRA
OPERATORS**

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Abstract. New fixed point theorems for maps (single-valued and multi-valued) between Fréchet spaces are presented. The proof relies on fixed point theory in Banach spaces and viewing a Fréchet space as the projective limit of a sequence of Banach spaces. In particular we obtain an applicable Leray-Schauder alternative, a Lefschetz fixed point theorem and a Krasnoselskii fixed point theorem in Fréchet spaces for general classes of maps.

Key Words and Phrases: fixed point theory, Fréchet space, single-valued and multivalued maps, projective limits.

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