

HYBRID TANGENTIAL PROPERTY AND COINCIDENCE POINT THEOREMS

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Abstract. Recently, Y. Liu et. al. [Common fixed points of single and multivalued maps, *Int. J. Math. Math. Sci.*, 19(2005), 3045-3055] defined common property $(E.A)$ for two pairs of hybrid maps and obtained some coincidence and fixed point results for such pairs. We define a new property for pairs of hybrid maps that contains common property $(E.A)$ and show that there is no need to use common property $(E.A)$ with full force and only our tangential condition works to prove Theorems 1.1 & 1.2.

Key Words and Phrases: fixed points, hybrid maps, property $(E.A)$, weak commutativity.

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1. INTRODUCTION AND PRELIMINARIES

Let X be a metric space with metric d . Then, for $x \in X$ and $A \subseteq X$, $d(x, A) = \inf\{d(x, y) : y \in A\}$. We denote by $CB(X)$ the class of all nonempty bounded closed subsets of X , by $CL(X)$ the class of all nonempty closed subsets of X . Let H be the generalized Hausdorff metric on $CL(X)$ generated by the metric d , that is,

$$H(A, B) = \max\{\sup_{x \in A} d(x, B), \sup_{y \in B} d(y, A)\}$$

for every $A, B \in CL(X)$. A point $p \in X$ is said to be a fixed point of $T : X \rightarrow CL(X)$ if $p \in Tp$. The point p is called a coincidence point of

$f : X \rightarrow X$ and $T : X \rightarrow CL(X)$ if $fp \in Tp$. The set of coincidence points of f and T is denoted by $C(f, T)$. Let Y be a nonempty subset of a normed space E . Then the set Y is called p -starshaped with $p \in Y$ if $\lambda x + (1 - \lambda)p \in Y$ for all $x \in Y$ and all real λ with $0 \leq \lambda \leq 1$. We define $T_\lambda x = \lambda Tx + (1 - \lambda)p$ and $[Tx, p] = \{T_\lambda x : \lambda \in [0, 1]\}$ for $T : Y \rightarrow CB(Y)$. Maps $f : X \rightarrow X$ and $T : X \rightarrow CL(X)$ are said to be compatible [10] if $fTx \in CL(X)$ for all $x \in X$ and $H(fTx_n, Tfx_n) \rightarrow 0$ whenever $\{x_n\}$ is a sequence in X such that $Tx_n \rightarrow A \in CL(X)$ and $fx_n \rightarrow t \in A$. Therefore, the maps $f : X \rightarrow X$ and $T : X \rightarrow CL(X)$ are non-compatible if $fTx \in CL(X)$ for all $x \in X$ and there exists at least one sequence $\{x_n\}$ in X such that $Tx_n \rightarrow A \in CL(X)$ and $fx_n \rightarrow t \in A$ but $\lim_{n \rightarrow \infty} H(fTx_n, Tfx_n) \neq 0$ or nonexistent. Maps $f : X \rightarrow X$ and $T : X \rightarrow CL(X)$ are weakly compatible [8] if they commute at their coincidence points, i.e., if $fTx = Tfx$ whenever $fx \in Tx$. Maps $f : X \rightarrow X$ and $T : X \rightarrow CL(X)$ are said to be (IT) -commuting [6, 18] at $x \in X$ if $fTx \subseteq Tfx$. Let $T : X \rightarrow CL(X)$, then $f : X \rightarrow X$ is said to be T -weakly [9] commuting at $x \in X$ if $ffx \in Tfx$. Maps $f : X \rightarrow X$ and $T : X \rightarrow CL(X)$ are said to satisfy the property $(E.A)$ [9] if there exists a sequence $\{x_n\}$ in X , some $t \in X$ and $A \in CL(X)$ such that $\lim_{n \rightarrow \infty} fx_n = t \in A = \lim_{n \rightarrow \infty} Tx_n$. Let $f, g : X \rightarrow X$ and $S, T : X \rightarrow CL(X)$, the hybrid pairs (f, T) and (g, S) are said to satisfy common property $(E.A)$ [11] if there exist two sequences $\{x_n\}, \{y_n\}$ in X , some $t \in X$, and $A, B \in CB(X)$ such that

$$\lim_{n \rightarrow \infty} Tx_n = A, \quad \lim_{n \rightarrow \infty} Sy_n = B, \quad \lim_{n \rightarrow \infty} fx_n = \lim_{n \rightarrow \infty} gy_n = t \in A \cap B. \quad (1)$$

Banach contraction principle is the most celebrated fixed point theorem. Nadler extended the Banach contraction principle to the case of multivalued maps. Afterward many authors obtained important fixed point theorems, see for example, Berinde [2, 3], Ćirić [4, 5], Petrusel [12, 14], Rus [13, 14] and Shahzad [16, 17]. Sastry and Murthy [15] defined the notion of tangential single-valued maps. Aamri and Moutawakil [1] rediscovered the notion of tangential maps and named it as property $(E.A)$. The class of maps satisfying property $(E.A)$ has remarkable property that it contain the class of compatible maps [7] as well as the class of non-compatible maps. Kamran [9] extended the notion of property $(E.A)$ to a hybrid pair (f, T) , introduced the notion of T -weak commutativity, and showed that (IT) -commutativity at a coincidence

point of a hybrid pair (f, T) implies T -weak commutativity at the same point but the converse is not true in general [9, Example 3.8]. Recently, Y. Liu et al. [11] defined common property $(E.A)$ for two pairs of hybrid maps and obtained the following results.

Theorem 1.1. [11] *Let f, g be two self-maps of the metric space (X, d) and let F, G be two maps from X into $CB(X)$ such that*

- (1) (f, F) and (g, G) satisfy common property $(E.A)$;
- (2) for all $x \neq y \in X$,

$$H(Fx, Gy) < \max \left\{ d(fx, gy), \frac{d(fx, Fx) + d(gy, Gy)}{2}, \frac{d(fx, Gy) + d(gy, Fx)}{2} \right\}. \quad (2)$$

If fX and gX are closed subsets of X , then

- (a) f and F have a coincidence point;
- (b) g and G have a coincidence point;
- (c) f and F have a common fixed point provided that f is F -weakly commuting at v and $ffv = fv$ for $v \in C(f, F)$;
- (d) g and G have a common fixed point provided that g is G -weakly commuting at v and $ggv = gv$ for $v \in C(g, G)$;
- (e) $f, g, F,$ and G have a common fixed point provided that both (c) and (d) are true.

Let $\phi : \mathbb{R}^+ \rightarrow \mathbb{R}^+$ be continuous and satisfy the following conditions:

- (i) ϕ is non-decreasing on \mathbb{R}^+
- (ii) $0 < \phi(t) < t$, for each $t \in (0, \infty)$.

Theorem 1.2. [11] *Let f, g be two self-maps of the metric space (X, d) and let F, G be two maps from X into $CB(X)$ such that*

- (1) (f, F) and (g, G) satisfy common property $(E.A)$;
- (2) for all $x \neq y \in X$,

$$H(Fx, Gy) \leq \phi(\max\{d(fx, gy), d(fx, Fx), d(gy, Gy), d(fx, Gy), d(gy, Fx)\}). \quad (3)$$

If fX and gX are closed subsets of X , then

- (a) f and F have a coincidence point;
- (b) g and G have a coincidence point;
- (c) f and F have a common fixed point provided that f is F -weakly commuting at v and $ffv = fv$ for $v \in C(f, F)$;
- (d) g and G have a common fixed point provided that g is G -weakly commuting at v and $g gv = gv$ for $v \in C(g, G)$;
- (e) f, g, F , and G have a common fixed point provided that both (c) and (d) are true.

The purpose of this note is to show that full force of common property (E.A) is not needed to prove the above results.

2. MAIN RESULTS

We have the following definition.

Definition 2.1. Let $f, g : X \rightarrow X$ and $S, T : X \rightarrow CL(X)$. The hybrid pair (f, T) is said to be g -tangential at $t \in X$ if there exist two sequences $\{x_n\}$, $\{y_n\}$ in X , $A \in CL(X)$ such that $\lim_{n \rightarrow \infty} S y_n \in CL(X)$ and

$$\lim_{n \rightarrow \infty} f x_n = \lim_{n \rightarrow \infty} g y_n = t \in A = \lim_{n \rightarrow \infty} T x_n. \quad (4)$$

Remark 2.2. Clearly, if the hybrid pairs (f, T) and (g, S) satisfy common property (E.A) then (f, T) is g -tangential whereas (g, S) is f -tangential. However, the following example shows that if (f, T) is g -tangential then it is not necessary that (f, T) and (g, S) satisfy common property (E.A).

Example 2.3. Let $X = [1, \infty)$ with the usual metric. Define $f, g : X \rightarrow X$ and $S, T : X \rightarrow CL(X)$ by

$$f x = 2 + \frac{1}{2}x, \quad g x = 2 + \frac{1}{3}x, \quad S x = [1, 2] \text{ and } T x = [2, 2 + x]$$

for all $x \in X$. Consider the sequences $\{x_n\} = \{1 + \frac{1}{n}\}_{n \in \mathbb{N}}$ and $\{y_n\} = \{\frac{3}{2} + \frac{1}{n}\}_{n \in \mathbb{N}}$ in X . Then,

$$\lim_{n \rightarrow \infty} f x_n = \lim_{n \rightarrow \infty} g y_n = \frac{5}{2} \in [2, 3] = \lim_{n \rightarrow \infty} T x_n.$$

Therefore, the hybrid pair (f, T) is g -tangential. Suppose that there exist $\{x_n\}$ and $\{y_n\}$ in X such that $\lim_{n \rightarrow \infty} T x_n = A$, $\lim_{n \rightarrow \infty} T y_n = B$ and $\lim_{n \rightarrow \infty} f x_n = \lim_{n \rightarrow \infty} g y_n = t \in A \cap B = \{2\}$. This implies that $\lim_{n \rightarrow \infty} x_n = \lim_{n \rightarrow \infty} y_n = 0$. Clearly, there exist no such sequences in X .

Now, we improve Theorems 1.1 & 1.2 by using above definition.

Theorem 2.4. *Let X be metric space with metric d , $f, g : X \rightarrow X$ and $S, T : X \rightarrow CL(X)$ be mappings such that*

$$H(Tx, Sy) \leq \max\{d(fx, gy), \alpha[d(fx, Tx) + d(gy, Sy)], \alpha[d(fx, Sy) + d(gy, Tx)]\}, \quad 0 \leq \alpha < 1, \quad (5)$$

for all $x, y \in X$ with $x \neq y$. If either (f, T) is g -tangential or (g, S) is f -tangential, and fX and gX are closed subsets of X , then $C(f, T) \neq \emptyset$ and $C(g, S) \neq \emptyset$. Further,

- (I) f and T have a common fixed point if f is T -weakly commuting at a and $ffa = fa$ where $a \in C(f, T)$;
- (II) g and S have a common fixed point if g is S -weakly commuting at b and $ggb = gb$ where $b \in C(g, S)$;
- (III) f, g, S and T have a common fixed point if (I) and (II) hold.

Proof. Suppose that the hybrid pair (f, T) is g -tangential, then there exist sequences $\{x_n\}$ and $\{y_n\}$ in X such that

$$\lim_{n \rightarrow \infty} fx_n = \lim_{n \rightarrow \infty} gy_n = t \in A = \lim_{n \rightarrow \infty} Tx_n. \quad (6)$$

Let $\lim_{n \rightarrow \infty} Sy_n = B$. We claim that $A = B$. Suppose not, i.e., $H(A, B) > 0$. Then, from (5), we have

$$\begin{aligned} H(A, Sy_n) &\leq H(A, Tx_n) + H(Tx_n, Sy_n) \\ &\leq H(A, Tx_n) + \max \left\{ d(fx_n, gy_n), \alpha[d(fx_n, Tx_n) + d(gy_n, Sy_n)], \right. \\ &\quad \left. \alpha[d(fx_n, Sy_n) + d(gy_n, Tx_n)] \right\} \\ &\leq H(A, Tx_n) + \max \left\{ d(fx_n, gy_n), \alpha[d(fx_n, Tx_n) + d(gy_n, A) \right. \\ &\quad \left. + H(A, Sy_n)], \alpha[d(fx_n, A) + H(A, Sy_n) + d(gy_n, Tx_n)] \right\}. \end{aligned}$$

Letting $n \rightarrow \infty$, we get

$$H(A, B) \leq \alpha H(A, B) < H(A, B), \quad (7)$$

which is a contradiction. Thus $\lim_{n \rightarrow \infty} Sy_n = A$. Since fX and gX are closed subsets of X , there exist a and b in X such that $\lim_{n \rightarrow \infty} fx_n = fa = t = gb =$

$\lim_{n \rightarrow \infty} gy_n$. We claim that $a \in C(f, T)$. If not, from (5), we have

$$H(Ta, Sy_n) \leq \max\{d(fa, gy_n), \alpha[d(fa, Ta) + d(gy_n, Sy_n)], \\ \alpha[d(fa, Sy_n) + d(gy_n, Ta)]\}.$$

Letting $n \rightarrow \infty$, we get

$$H(Ta, A) \leq \alpha d(fa, Ta) < d(fa, Ta).$$

Using the definition of Hausdorff metric, the above inequality gives $d(fa, Ta) < d(fa, Ta)$, a contradiction. Thus $C(f, T) \neq \emptyset$. Now we show that $C(g, S) \neq \emptyset$. From (5), we get

$$\begin{aligned} d(gb, Sb) &= d(fa, Sb) \\ &\leq H(Ta, Sb) \\ &\leq \max\{d(fa, gb), \alpha[d(fa, Ta) + d(gb, Sb)], \\ &\quad \alpha[d(fa, Sb) + d(gb, Ta)]\} \\ &= \alpha d(gb, Sb) \\ &< d(gb, Sb), \end{aligned}$$

which proves that $b \in C(g, S)$, since Sb is closed. The proof is similar if g -tangentiality of (f, T) is replaced by f -tangentiality of (g, S) . Further, by (I), we have $ffa = fa$ and $ffa \in Tfa$. Thus $t = ft \in Tt$. This proves (I). A similar argument proves (II). Then (III) is immediate.

Theorem 2.5. *Let X be metric space with metric d , $f, g : X \rightarrow X$ and $S, T : X \rightarrow CL(X)$ be mappings such that*

$$H(Tx, Sy) \leq \tag{8}$$

$\leq \max\{d(fx, gy), \phi(d(fx, Tx)) + \phi(d(gy, Sy)), \phi(d(fx, Sy)) + \phi(d(gy, Tx))\}$,
for all $x, y \in X$ with $x \neq y$. If either (f, T) is g -tangential or (g, S) is f -tangential and fX and gX are closed subsets of X , then $C(f, T) \neq \emptyset$ and $C(g, S) \neq \emptyset$. Further,

- (I) f and T have a common fixed point if f is T -weakly commuting at a and $ffa = fa$ where $a \in C(f, T)$;
- (II) g and S have a common fixed point if g is S -weakly commuting at b and $ggb = gb$ where $b \in C(g, S)$;

(III) f, g, S and T have a common fixed point if (I) and (II) hold.

Proof. Suppose that the hybrid pair (f, T) is g -tangential, then there exist sequences $\{x_n\}$ and $\{y_n\}$ in X such that

$$\lim_{n \rightarrow \infty} fx_n = \lim_{n \rightarrow \infty} gy_n = t \in A = \lim_{n \rightarrow \infty} Tx_n. \tag{9}$$

Let $\lim_{n \rightarrow \infty} Sy_n = B$. We claim that $A = B$. Suppose not, i.e., $H(A, B) > 0$. Then, from (8), we have

$$\begin{aligned} H(A, Sy_n) &\leq H(A, Tx_n) + H(Tx_n, Sy_n) \\ &\leq H(A, Tx_n) + \max\{d(fx_n, gy_n), \phi(d(fx_n, Tx_n)) + \phi(d(gy_n, Sy_n)), \\ &\quad \phi(d(fx_n, Sy_n)) + \phi(d(gy_n, Tx_n))\} \\ &\leq H(A, Tx_n) + \max\{d(fx_n, gy_n), \phi(d(fx_n, Tx_n)) + \phi(d(gy_n, A) \\ &\quad + H(A, Sy_n)), \phi(d(fx_n, A) + H(A, Sy_n)) + \phi(d(gy_n, Tx_n))\}. \end{aligned}$$

Letting $n \rightarrow \infty$, we get

$$H(A, B) \leq \phi(H(A, B)) < H(A, B), \tag{10}$$

which is a contradiction. Thus $\lim_{n \rightarrow \infty} Sy_n = A$. Since fX and gX are closed subsets of X , there exist a and b in X such that $\lim_{n \rightarrow \infty} fx_n = fa = t = gb = \lim_{n \rightarrow \infty} gy_n$. We claim that $a \in C(f, T)$. If not, from (8), we have

$$\begin{aligned} H(Ta, Sy_n) &\leq \max\{d(fa, gy_n), \phi(d(fa, Ta)) + \phi(d(gy_n, Sy_n)), \\ &\quad \phi(d(fa, Sy_n)) + \phi(d(gy_n, Ta))\}. \end{aligned}$$

Letting $n \rightarrow \infty$, we get

$$H(Ta, A) \leq \phi(d(fa, Ta)) < d(fa, Ta).$$

Using the definition of Hausdorff metric, the above inequality gives $d(fa, Ta) < d(fa, Ta)$, a contradiction. Thus $C(f, T) \neq \emptyset$. Now we show

that $C(g, S) \neq \emptyset$. From (8), we get

$$\begin{aligned} d(gb, Sb) &= d(fa, Sb) \\ &\leq H(Ta, Sb) \\ &\leq \max\{d(fa, gb), \phi(d(fa, Ta)) + \phi(d(gb, Sb)), \\ &\quad \phi(d(fa, Sb)) + \phi(d(gb, Ta))\} \\ &= \phi(d(gb, Sb)) \\ &< d(gb, Sb). \end{aligned}$$

which proves that $b \in C(g, S)$, since Sb is closed. The proof is similar if g -tangentiality of (f, T) is replaced by f -tangentiality of (g, S) . Further, by (I), we have $ffa = fa$ and $ffa \in Tfa$. Thus $t = ft \in Tt$. This proves (I). A similar argument proves (II). Then (III) is immediate.

Remark 2.6. *The conclusions of Theorems 2.4 & 2.5 are still valid if the closedness of fX and gX are replaced by either of the following two conditions;*

- (i) fX is closed subset of X and $TX \subseteq gX$,
- (ii) gX is closed subset of X and $SX \subseteq fX$.

As an application of our result, following Shahzad [17], we have the following.

Theorem 2.7. *Let Y be a closed and p -starshaped subset of normed space E with $p \in Y$, $f, g : Y \rightarrow Y$, and $S, T : Y \rightarrow CB(Y)$. Suppose that*

$$\begin{aligned} H(Tx, Sy) \leq \phi(\max\{\|fx, gy\|, d(fx, [Tx, p]), d(gy, [Sy, p]), \\ d(fx, [Sy, p]), d(gy, [Tx, p])\}), \end{aligned} \quad (11)$$

for all $x, y \in Y$. If either (f, T_λ) is g -tangential or (g, S_λ) is f -tangential for each $0 \leq \lambda \leq 1$; here $T_\lambda x = \lambda Tx + (1 - \lambda)p$ and $S_\lambda x = \lambda Sx + (1 - \lambda)p$. If $(f - T)(Y)$ and $(g - S)(Y)$ are closed, $T(Y)$ and $S(Y)$ are bounded, and $fY = Y = gY$ then $C(f, T) \neq \emptyset$ and $C(g, S) \neq \emptyset$. Further,

- (I) f and T have a common fixed point if f is T -weakly commuting at a and $ffa = fa$ where $a \in C(f, T)$;
- (II) g and S have a common fixed point if g is S -weakly commuting at b and $ggb = gb$ where $b \in C(g, S)$;
- (III) f, g, S and T have a common fixed point if (I) and (II) hold.

Proof. For each n and $x \in Y$, define $S_n, T_n : Y \rightarrow CB(Y)$ by $T_n x = \lambda_n T x + (1 - \lambda_n)p$, $S_n x = \lambda_n S x + (1 - \lambda_n)p$, where $\{\lambda_n\}$ is a sequence in $(0, 1)$ such that $\lambda_n \rightarrow 1$ as $n \rightarrow \infty$. Observe that for each n , $S_n(Y)$ and $T_n(Y)$ are subsets of Y , since Y is p -starshaped. From inequality (11), we get

$$\begin{aligned} H(T_n x, S_n y) &= \lambda_n H(T x, S y) \\ &\leq \lambda_n \phi(\max\{\|f x, g y\|, d(f x, [T x, p]), d(g y, [S y, p]), \\ &\quad d(f x, [S y, p]), d(g y, [T x, p])\}) \\ &\leq \phi(\max\{\|f x, g y\|, d(f x, T_n x), d(g y, S_n y), d(f x, S_n y), \\ &\quad d(g y, T_n x)\}), \end{aligned}$$

for all $x, y \in Y$. Now it follows from Theorem 2.5 that there are $x_n, z_n \in Y$ such that $f x_n \in T_n x_n$ and $g z_n \in S_n z_n$. Therefore, there exist $y_n \in T x_n$ and $t_n \in S z_n$ such that $f x_n - y_n = (1 - \lambda_n)(p - y_n)$ and $g z_n - t_n = (1 - \lambda_n)(p - t_n)$. Since $T(Y)$ and $S(Y)$ are bounded, it follows that $f x_n - y_n \rightarrow 0$ and $g z_n - t_n \rightarrow 0$ as $n \rightarrow \infty$. The closedness of $(f - T)(Y)$ and $(g - S)(Y)$ further implies that $0 \in (f - T)(Y)$ and $0 \in (g - S)(Y)$. Hence $C(f, T) \neq \emptyset$ and $C(g, S) \neq \emptyset$. So, $f a \in T a$ and $g b \in S b$ for some $a, b \in Y$.

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REFERENCES

- [1] M. Aamri and D. El Moutawakil, *Some new common fixed point theorems under strict contractive conditions*, J. Math. Anal. Appl., **270**(2002), 181-188.
- [2] V. Berinde, *Approximating fixed points of weak contractions using the Picard iteration*, Nonlinear Analysis Forum, **9**(2004), No. 1, 43-53.
- [3] M. Berinde and V. Berinde, *On general class of multi-valued weakly Picard mappings*, J. Math. Anal. Appl., **326**(2007), 772-782.
- [4] Lj. B. Ćirić, *Generalized contractions and fixed-point theorems*, Pub. Inst. Math., **12**(1971), 19-26.
- [5] Lj. B. Ćirić, *Fixed point theory, Contraction mapping principle*, FME Press, Beograd, 2003.
- [6] S. Itoh, and W. Takahashi, *Single valued mappings, multivalued mappings and fixed point theorems*, J. Math. Anal. Appl., **59**(1977), 514-521.
- [7] G. Jungck, *Compatible mappings and common fixed points*, Internat. J. Math. Math. Sci., **9**(1986), 771-779.

- [8] G. Jungck and B. E. Rhoades, *Fixed points for set valued functions without continuity*, Indian J. Pure Appl. Math., **29**(1998), 227-238.
- [9] T. Kamran, *Coincidence and fixed points for hybrid strict contractions*, J. Math. Anal. Appl., **299**(2004) 235-241.
- [10] H. Kaneko and S. Sessa, *Fixed point theorems for compatible multivalued and single valued mappings*, Internat. J. Math and Math. Sci., **12**(1989), 257-262.
- [11] Y. Liu, J. Wu, Z. Li, *Common fixed points of single and multivalued maps*, Int. J. Math. Math. Sci., **19**(2005), 3045-3055.
- [12] A. Petrusel, *Operatorial inclusions*, House of the Book of Science, Cluj-Napoca, 2002.
- [13] I.A. Rus, *Teoria Punctului Fix*, II, Univ. Babeş-Bolyai, Cluj, 1973.
- [14] I. A. Rus, A. Petrusel and A. Sintamarian, *Data dependence of fixed point set of some multi-valued weakly Picard operators*, Nonlinear Anal., **52**(2003), 1947-1959.
- [15] K. P. R. Sastry, I. R. S. Krishna Murthy, *Common fixed points of two partially commuting tangential selfmaps on metric space*, J. Math. Anal. Appl., **250**(2000), 731-734.
- [16] N. Shahzad, *Coincidence points and R-subweakly commuting multivalued maps*, Demonstratio Math., **36**(2003), 427-431.
- [17] D. O'Regan and N. Shahzad, *Coincidence points and invariant approximation results for multimaps*, Acta Mathematica Sinica, **23**(2007), 1601-1610.
- [18] S. L. Singh and S. N. Mishra, *Coincidence and fixed points of nonself hybrid contractions*, J. Math. Anal. Appl., **256**(2001), 486-497.

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