

FIXED POINTS OF A SEQUENCE OF ASYMPTOTICALLY NONEXPANSIVE MAPPINGS

SHUECHIN HUANG

Department of Applied Mathematics

Dong Hwa University

Hualien 97401, Taiwan

E-mail: shuang@mail.ndhu.edu.tw

Abstract. In a Banach space with uniformly Gâteaux differentiable norm and uniform normal structure, by constructing an iterative sequence $\{z_n\}$ for a family of asymptotically nonexpansive mappings $\{T_m\}_{m=1}^\infty$, the necessary and sufficient conditions for the strong convergence of $\{z_n\}$ to a common fixed point of $\{T_m\}_{m=1}^\infty$ are analyzed.

Key Words and Phrases: Asymptotically nonexpansive mapping, normalized duality mapping, fixed point, weakly sequential continuity, demiclosedness, uniformly Gâteaux differentiable norm, uniform normal structure, uniformly convex Banach space.

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