

ISHIKAWA ITERATIONS FOR EQUILIBRIUM AND FIXED POINT PROBLEMS FOR NONEXPANSIVE MAPPINGS IN HILBERT SPACES

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Abstract. In this paper, we introduce an iterative scheme Ishikawa-type for finding a common element of the set $EP(G)$ of the equilibrium points of a bifunction G and the set $Fix(T)$ of fixed points of a nonexpansive mapping T in a Hilbert space H . We prove that the method converges strongly to an element $z \in Fix(T) \cap EP(G)$ which is the unique solution of the variational inequality $\langle (A - \gamma f)z, x - z \rangle \geq 0$ for every $x \in Fix(T) \cap EP(G)$. The results presented here are situated on the line of research of [5, 6, 7, 10, 12, 13].

Key Words and Phrases: equilibrium problem, fixed points, nonexpansive mappings, variational inequality, Ishikawa iterations.

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