

## FUNCTIONAL DIFFERENTIAL EQUATIONS WITH FRACTIONAL ORDER AND INFINITE DELAY

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**Abstract.** In this paper, we shall establish sufficient conditions for the existence of mild solutions for some densely defined semilinear functional and neutral functional differential equations with fractional order and infinite delay. Our approach is based on a nonlinear alternative of Leray-Schauder type.

**Key Words and Phrases:** semilinear functional differential equation, fractional derivative, fractional integral, fixed point, semigroups, mild solutions.

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