

UNIQUENESS RESULTS FOR PARTIAL FUNCTIONAL DIFFERENTIAL EQUATIONS IN FRÉCHET SPACES

SELMA BAGHLI* AND MOUFFAK BENCHOHRA**

*Laboratoire de Mathématiques,
Université de Sidi Bel-Abbès,
BP 89, 22000 Sidi Bel Abbès, Algérie
E-mail: selma_baghli@yahoo.fr

**Laboratoire de Mathématiques,
Université de Sidi Bel-Abbès,
BP 89, 22000 Sidi Bel Abbès, Algérie
E-mail: benchohra@univ-sba.dz

Abstract. We are giving, in this paper, the existence of the unique mild solution on a semi-infinite interval for a first order semilinear functional differential equation using a recent nonlinear Frigon-Granas alternative for contractions in Fréchet spaces, combined with semigroup theory.

Key Words and Phrases: semilinear evolution equation, mild solution, fixed-point theory, nonlinear alternative, semigroup theory, nonlocal conditions, Fréchet spaces.

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