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APPROXIMATION OF THE SOLUTIONS OF THE DARBOUX PROBLEM FOR THIRD ORDER HYPERBOLIC INCLUSIONS

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Abstract. In this paper we consider the Darboux problem for a third order hyperbolic inclusion of the form $u_{xyz} \in F(x, y, z, u)$. Using the notion of uniform convergence on compact domains as defined by Arrigo Cellina for a sequence of single-valued functions $f_k : \Lambda \to \mathbb{R}^n$ such that $f_k \to F$, where F is a multifunction, it is considered a sequence of approximating univalued equations of the form $u_{xyz} = f_k(x, y, z, u)$ and it is proved that they have a unique solution based on Schauder's Fixed Point Theorem. Using a characterization theorem for the solutions of the Darboux Problem for the specified inclusion, it is proved that the sequence of solutions to the univalued equations uniformly converges, on compact sets, to a solution of the Darboux Problem for the considered inclusion.

Key Words and Phrases: multifunction, hyperbolic inclusion, upper semi-continuity, initial values, absolutely continuous in Carathéodory's sense function, Aumann integral, uniform convergence of a sequence of single-valued functions on compact sets to a multifunction.
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