

## ABSTRACT MODELS OF STEP METHOD WHICH IMPLY THE CONVERGENCE OF SUCCESSIVE APPROXIMATIONS

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**Abstract.** This paper has three goals:

- to present two abstract models: forward step model and backward step model;
- to prove that the global operator which appear in these models are weakly Picard;
- to give applications to functional differential equations with retarded argument and to functional differential equations with advanced argument.

**Key Words and Phrases:** operators on cartesian product, step method, Picard operators, weakly Picard operators, functional differential equations, retarded argument, advanced argument.

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