Fixed Point Theory, Volume 9, No. 1, 2008, 207-220
http://www.math.ubbcluj.ro/~nodeacj/sfptcj.html

FUNCTIONAL-DIFFERENTIAL EQUATIONS WITH MAXIMA OF MIXED TYPE

DIANA OTROCOL* AND IOAN A. RUS**

*Tiberiu Popoviciu Institute of Numerical Analysis
Romanian Academy
Cluj-Napoca, Romania
E-mail: dotrocol@ictp.acad.ro

**Department of Applied Mathematics
Babeş-Bolyai University
Cluj-Napoca, Romania
E-mail: iarus@math.ubbcluj.ro

Abstract. In this paper we study the following second order functional-differential equations with maxima, of mixed type,

$$-x''(t) = f(t, x(t), \max_{t-h_1 \leq \xi \leq t} x(\xi), \max_{t \leq \xi \leq t+h_2} x(\xi)),\ t \in [a, b]$$

with "boundary" conditions

$$\begin{cases} x(t) = \varphi(t),\ t \in [a - h_1, a], \\ x(t) = \psi(t),\ t \in [b, b + h_2]. \end{cases}$$

The plan of the paper is the following: 1. Introduction 2. Picard and weakly Picard operator 3. The operator $\max_I$ 4. Existence and uniqueness 5. Inequalities of Čaplygin type 6. Data dependence: monotony 7. Data dependence: continuity 8. Examples.

Key Words and Phrases: Picard operator, weakly Picard operators, equation of mixed type, equations with maxima, fixed points, data dependence.
2000 Mathematics Subject Classification: 34K10, 47H10.

References

[1] N.V. Azbelev (ed.), *Functional-Differential Equations*, Perm. Politekh. Inst. Perm, 1985 (in Russian).

[2] D. Bainov, D. Mishev, *Oscillation Theory of Operator-Differential Equations*, World Scientific, Singapore, 1995.

[3] D.D. Bainov, V. A. Petrov, V.S. Proyteheva, *Existence and asymptotic behavior of nonoscillatory solutions of second order neutral differential equations with "maxima"*, J. Comput. Appl. Math., **83**(1997), no. 2, 237-249.

[4] L. Georgiev, V.G. Angelov, *On the existence and uniqueness of solutions for maximum equations*, Glas. Mat., **37**(2002), 275-281.

[5] J. Hale, *Theory of Functional Differential Equations*, Springer, 1977.

[6] T. Jankovskyi, *System of differential equations with maxima*, Dopov, Akad. Nauk. Ukr., **8**(1997), 57-60.

[7] V. Kolmanovskii, A. Myshkis, *Applied Theory of Functional Differential Equations*, Kluwer Acad. Publ., Dordrecht, 1992.

[8] E. Liz, *Monotone iterative technique in ordered Banach spaces*, Nonlinear Analysis, **30**(1997), 5179-5190.

[9] A.R. Magomedov, *Existence and uniqueness theorems for solutions of differential equations with maxima containing a functional parameter*, Arch. Math., Brne, **28**(1992), 139-154 (in Russian).

[10] I.A. Rus, *Picard operators and applications*, Scientiae Mathematicae Japonicae, **58**(2003), 191-219.

[11] I.A. Rus, *Functional differential equations of mixed type, via weakly Picard operators*, Seminar on Fixed Point Theory Cluj-Napoca, **3**(2002), 335-346.

[12] I.A. Rus, *Generalized Contractions and Applications*, Cluj University Press, 2001.

[13] I.A. Rus, *Weakly Picard operators and applications*, Seminar on Fixed Point Theory Cluj-Napoca, **2**(2001), 41-58.

[14] I.A. Rus, A. Petruşel, M. Şerban, *Weakly Picard operators: equivalent definitions, applications and open problems*, Fixed Point Theory, **7**(2006), 3-22.

[15] E. Stepanov, *On solvability of same boundary value problems for differential equations with "maxima"*, Topological Methods in Nonlinear Analysis, **8**(1996), 315-326.

[16] B.G. Zhang, G. Zhang, *Qualitative properties of functional equations with "maxima"*, Rocky Mount. J. Math., **29**(1999), 357-367.

[17] M. Zima, *Applications of the spectral radius to same integral equations*, Comment. Math. Univ. Caroline, **36**(1997), 695-703.

Received: December 18, 2007; Accepted: February 10, 2008.