# ZBĂGANU CONSTANT AND NORMAL STRUCTURE

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**Abstract.** The aim of this paper is to give sufficient conditions for the normal structure of a Banach space, involving the so-called Zbăganu constant $C_Z(X)$ which is a quadratic one introduced in [8] in order to characterize the inner product spaces.

**Key Words and Phrases**: Normal structure, fixed point property, von Neumann-Jordan constant, Zbaganu constant.

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