

## NEW RESULTS ABOUT SOME NONLINEAR OPERATORS

G. ISAC

Department of Mathematics  
Royal Military College of Canada  
P. O. Box 17000 STN Forces  
Kingston, Ontario, Canada K7K 7B4  
E-mail: gisac@cogeco.ca

**Abstract.** We present in this paper three new classes of nonlinear operators related to the study of complementarity problems and variational inequalities. Some open subjects related to these operators are also put in evidence.

**Key Words and Phrases:**  $B$ -quasi-bounded operators, scalarly compact operators and normal operators in infinite dimensional spaces.

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