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ON THE MULTIPLICITY OF THE CIRCUMFERENCE IN PLANAR POLYNOMIAL VECTOR FIELDS

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Abstract. In this work we consider planar polynomial differential systems of the form:

$$\dot{x} = P(x, y), \quad \dot{y} = Q(x, y),$$

where $P(x, y)$ and $Q(x, y)$ are polynomials with real coefficients whose maximum degree is d. We only consider systems of this form with the circumference $x^2 + y^2 - 1 = 0$ as a periodic orbit. These systems take the form:

$$\dot{x} = -y\, c(x, y) + f(x, y)\, a(x, y), \quad \dot{y} = x\, c(x, y) + f(x, y)\, b(x, y),$$

where $f(x, y) = (x^2 + y^2 - 1)/2$ and a, b and c are real polynomials. Our interest in this work is to study the multiplicity of the circumference as periodic orbit of the aforementioned system. This work contains some theorems that characterize when the circumference is a limit cycle of multiplicity m and when it belongs to a period annulus. Moreover, if we assume that the system is of a particular form, we will give an upper bound for the possible multiplicities that the circumference may have as a limit cycle. Finally, we apply our results to some examples.

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References

[1] A. A. Andronov, et al., *Qualitative theory of second-order dynamic systems.* Halsted Press (A division of John Wiley and Sons), New York-Toronto, Ont., Israel Program for Scientific Translations, Jerusalem-London, 1973.

[2] V.I. Arnold, *Loss of stability of self-induced oscillations near resonance, and versal deformations of equivariant vector fields*, Funct. Anal. Appl., **11**(1977) 85-92.

[3] D. Cerveau, A. Lins Neto, *Holomorphic foliations in* $C\mathrm{P}(2)$ *having an invariant algebraic curve*, Ann. Inst. Fourier (Grenoble), **41**(1991), no. 4, 883-903.

[4] J. Chavarriga, J. Llibre and J. Sorolla, *Algebraic limit cycles of degree* 4 *for quadratic systems*, J. Differential Equations, **200**(2004), 206-244.

[5] C. Chicone, *Ordinary Differential Equations with Applications*, Springer-Verlag New-York, 1999.

[6] Yuan-shün Ch'in, *On algebraic limit cycles of degree* 2 *of the differential equation*

$$\frac{dy}{dx} = \frac{\sum_{0\leqq i+j\leqq 2} a_{ij}x^i y^j}{\sum_{0\leqq i+j\leqq 2} b_{ij}x^i y^j},$$

Sci. Sinica, **7** (1958), 934-945, and Acta Math. Sinica, **8**(1958), 23-35.

[7] C. Christopher, J. Llibre, C. Pantazi, X. Zhang, *Darboux integrability and invariant algebraic curves for planar polynomial systems*, J. Phys. Ser. A, **35**(2002), no. 10, 2457-2476.

[8] C. Christopher, J. Llibre, C. Pantazi and S. Walcher. Inverse problems for invariant algebraic curves: explicit computations. Preprint, 2008.

[9] J.-P. Françoise, C.C. Pugh, *Keeping track of limit cycles,* J. Differential Equations, **65**(1986), no. 2, 139-157.

[10] J.-P. Françoise, *Successive derivatives of a first return map, application to the study of quadratic vector fields,* Ergodic Theory Dynam. Systems, **16**(1996), no. 1, 87-96.

[11] J.-P. Françoise, *The successive derivatives of the period function of a plane vector field,* J. Differential Equations, **146**(1998), no. 2, 320-335.

[12] W. Fulton, *Algebraic curves. An Introduction to Algebraic Geometry,* Mathematics Lecture Notes Series, W. A. Benjamin, Inc., New York-Amsterdam, 1969.

[13] A. Gasull, J. Giné and M. Grau, *Multiplicity of limit cycles and analytic m-solutions for planar differential systems*, J. Differential Equations, **240**(2007), 375-398.

[14] A. Gasull, J. Torregrosa, *A new approach to the computation of the Lyapunov constants. The geometry of differential equations and dynamical systems,* Comput. Appl. Math., **20**(2001), no. 1-2, 149-177.

[15] H. Giacomini, M. Grau, *On the stability of limit cycles for planar differential systems,* J. Differential Equations, **213**(2005), no. 2, 368-388.

[16] J. K. Hale, *Ordinary differential equations*, Pure and Applied Mathematics, Vol. XXI. Wiley-Interscience (John Wiley & Sons), New York-London-Sydney, 1969.

[17] N. G. Lloyd, *A note on the number of limit cycles in certain two-dimensional systems,* J. London Math. Soc., (2) **20**(1979), no. 2, 277-286.

[18] P. Mardešić, *An explicit bound for the multiplicity of zeros of generic Abelian integrals,* Nonlinearity, **4**(1991), no. 3, 845-852.

[19] L. Perko, *Differential Equations and Dynamical Systems*, Third edition, Texts in Applied Mathematics, **7**, Springer-Verlag, New York, 2001.

[20] J. Sotomayor, *Liçoes de equaçoes diferenciais ordinárias*, Projeto Euclides, **11**, Instituto de Matemática Pura e Aplicada, Rio de Janeiro, 1979.

[21] A.N. Varchenko, *Estimation of the number of zeros of an abelian integral depending on a parameter, and limit cycles,* Funktsional. Anal. i Prilozhen., **18**(1984), no. 2, 14-25.

[22] Y. Q. Ye et al., *Theory of limit cycles*, Second edition. Transl. Math. Monographs, Vol. **66**, Amer. Math. Soc., Providence, RI, 1986.

[23] Z. Zhang et al., *Qualitative theory of differential equations,* Transl. Math. Monographs, Vol. **101**, Amer. Math. Soc., Providence, RI, 1992.

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