

## WEAK CONVERGENCE THEOREM BY A MODIFIED EXTRAGRADIENT METHOD FOR NONEXPANSIVE MAPPINGS AND MONOTONE MAPPINGS

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**Abstract.** In this paper, we introduce a modified extragradient method for finding the common element of the set of fixed points of a nonexpansive mapping and the set of solutions of the variational inequality problem for a monotone and Lipschitz continuous mapping. We obtain a weak convergent theorem for two sequences generated by this modified extragradient method.

**Key Words and Phrases:** Modified extragradient method, fixed points, monotone mappings, nonexpansive mappings, variational inequalities.

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