

COMMON FIXED POINT THEOREMS FOR ASYMPTOTICALLY I -CONTRACTIVE MAPPINGS WITHOUT CONVEXITY

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Abstract. This paper aims to present common fixed point theorems for I -nonexpansive mappings from (I, T) -star shaped subset of a uniformly convex Banach space X into X under some asymptotic I -contraction assumptions. These results extend and generalize results valid for bounded convex sets or asymptotically compact sets.

Key Words and Phrases: Asymptotic, asymptotic cone, I -contraction, weakly compatible maps, derivative at infinity, firm asymptotic cone, common fixed point, I -nonexpansive map.

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REFERENCES

- [1] A. Agadi and J.-P. Penot, *Asymptotic approximation of sets with application in mathematical programming*, Preprint, Univ. of Pau, February 1996.
- [2] F.E. Browder, *Nonexpansive nonlinear operators in a Banach space*, Proc. Nat. Acad. Sci., **54**(1965), 1041-1044.
- [3] F.E. Browder, *Nonlinear operators and nonlinear equations of evolution in Banach spaces*, Proc. Symp. Pure Math. Vol. 18, Amer. Math. Soc. Providence, 1976.

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- [4] J.P. Dedieu, *Cone asymptotic d'un ensemble non convexe. Application à l'optimisation*, C.R. Acad. Sci. Paris, **287**(1977), 501-503.
- [5] K. Goebel and W.A. Kirk, *Topics in Metric Fixed Point Theory*, Cambridge University Press, Cambridge, UK, 1990.
- [6] D. Göhde, *Zum prinzip der kontaktiven abbildung*, Math. Nach., **30**(1965), 251-258.
- [7] G. Jungck, *Commuting maps and fixed points*, Amer. Math. Monthly, **83**(1976), 261-263.
- [8] W. A. Kirk, *A fixed-point theorem for mappings which do not increase distances*, Amer. Math. Monthly, **72**(1965), 1004-1006.
- [9] W. A. Kirk, *Nonexpansive mappings and asymptotic regularity*, Nonlinear Anal. TMA, **40A**(2000), 323-332.
- [10] W.A. Kirk, *The fixed point property and mappings which are eventually nonexpansive*, in Kartsatos, Athanassios G. (ed.), *Theory and applications of nonlinear operators of accertive and monotone type*, Lect. Notes Pure Appl. Math., 178 Marcel Dekker, New York 1996, 141-147.
- [11] W.A. Kirk, C. Martinez-Yanez and S.S. Shin, *Asymptotically nonexpansive mappings*, Nonlinear Anal. TMA, **33**(1998), 1-12.
- [12] M.A. Krasnoselski, *Positive Solutions of Operator Equations*, Noordhoff, Groningen (1964).
- [13] D.T. Luc, *Recession maps and applications*, Optimization **27**(1993), 1-15.
- [14] D.T. Luc, *Recessively compact sets: uses and properties*, Set-Valued Anal., **10**(2002), 15-35.
- [15] D.T. Luc and J.-P. Penot, *Convergence of asymptotic directions*, Trans. Amer. Math. Soc., **352**(2001), 4095-4121.
- [16] J.-P. Penot, *Fixed point theorems without convexity*, Memoire Soc. Math. de France, **60**(1977), 129-152.
- [17] J.-P. Penot, *Compact nets, filters and relations*, J. Math. Anal. Appl., **93**(1983), 400-417.
- [18] J.-P. Penot, *What is quasiconvex analysis*, Optimization, **47**(2000), 35-100.
- [19] J.-P. Penot, *A metric approach to asymptotic analysis*, Bull. Sci. Math., **127**(2003), 815-833.
- [20] J.-P. Penot, *A fixed point theorem for asymptotically contractive mappings*, Proc. Amer. Math. Soc., **131**(2003), 2371-2377.
- [21] J.-P. Penot, C. Zalinescu, *Continuity of usual operations and variational convergences*, Preprint, Univ. of Pau, 2000 and 2001.
- [22] B.D. Rouhani and W.A. Kirk, *Asymptotic properties of nonexpansive iterations in reflexive spaces*, J. Math. Anal. Appl., **236**(1999), 281-289.
- [23] C. Zalinescu, *Recession cones and asymptotically compact sets*, J. Optim. Theory Appl., **77**(1993), 209-220.
- [24] E. Zeidler, *Nonlinear Functional Analysis and Applications, Part 1: Fixed -Point Theorems*, Springer Verlag, New York (1986).

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