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FIXED POINT APPROACH TO SOME TWO-POINT BOUNDARY VALUE PROBLEMS FOR DIFFERENTIAL INCLUSIONS ON MANIFOLDS

YURI E. GLIKLIKH* AND PETER S. ZYKOV**

* Mathematics Faculty, Voronezh State University,
Universitetskaya pl. 1, 394006 Voronezh, Russia
E-mail: yeg@math.vsu.ru

** Physics and Mathematics Faculty, Kursk State University,
ul. Radishcheva 33, 305416 Kursk, Russia
petya39b@mail.ru

Abstract. We investigate the two-point boundary value problem for second order differential inclusions of the form $\frac{D}{dt}\dot{m}(t) \in F(t, m(t), \dot{m}(t))$ on a complete Riemannian manifold for a couple of points, non-conjugate along at least one geodesic of Levi-Civitá connection, where $\frac{D}{dt}$ is the covariant derivative of Levi-Civitá connection and $F(t, m, X)$ is convex-valued and satisfies the upper Carathéodory condition or is almost lower semi-continuous set-valued vector field such that $\|F(t, m, X)\| < a(t, m)\|X\|^2$ with continuous $a(t, m) > 0$. Some conditions on certain geometric characteristics, on the distance between points and on $a(t, m)$ are found, under which the problem is solvable on any time interval. The solution is constructed from a fixed point of a certain integral-type operator, acting in the space of continuous curves in the tangent space at initial point. The existence of fixed point is proved by application of Bohnenblust-Karlin and Schauder theorems.

Key Words and Phrases: Second order differential inclusion, complete Riemannian manifold, quadratic growth, two-point boundary value problem, set-valued map, fixed point.

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