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# ON SOME APPROXIMATION METHODS IN THE THEORY OF OPERATOR INCLUSIONS

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Dedicated to Professor Ioan A. Rus on the occasion of his 70th birthday

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**Abstract.** In the present paper we use approximation methods for the study of operator inclusions of the form $a(x) \in \Phi(x)$, where $a$ is a closed linear surjective operator from a Banach space onto another one, and $\Phi$ is a multimap being a composition of a multimap with "good" values and a continuous singlevalued map. As application we consider the solvability of an integro-differential system which may be treated as a control object with an integral feedback.

**Key Words and Phrases**: multivalued map, fixed point, coincidence point, continuous selection, operator inclusion, closed linear operator, integro-differential system.
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