

A FIXED POINT PROOF OF THE CONVERGENCE OF A NEWTON-TYPE METHOD

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Abstract. By applying an appropriate fixed point technique, it is shown that a certain Newton-type iterative method converges to the unique solution of the scalar nonlinear equation $f(x) = 0$, under weak smoothness conditions, involving only the function f and its first derivative f' . For this Newton-like method, an error estimate, better than the one known in the case of the classical Newton method, is also established.

Key Words and Phrases: 65H05; 47J25; 47H09; 47H10.

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