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LOSSY TRANSMISSION LINES TERMINATED BY NONLINEAR R-LOADS - PERIODIC REGIMES

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Dedicated to Professor Ioan A. Rus on the occasion of his 70th birthday

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Abstract. The paper is devoted to the analysis of lossy transmission lines terminated by nonlinear R-loads whose V-I characteristic is approximated by polynomials. By means of fixed point method conditions for the existence of periodic solutions are formulated. A complete exposition of reducing of the mixed problem for lossy transmission lines to an initial value problem for a neutral functional differential equation on the boundary is presented. An example demonstrates the applicability of the method.It is shown of how to choose the relations between basic quantities in the process of analysis and design of the transmission lines.

Key Words and Phrases: lossy transmission lines, nonlinear R-load, polynomial nonlinearities, fixed point theorem.
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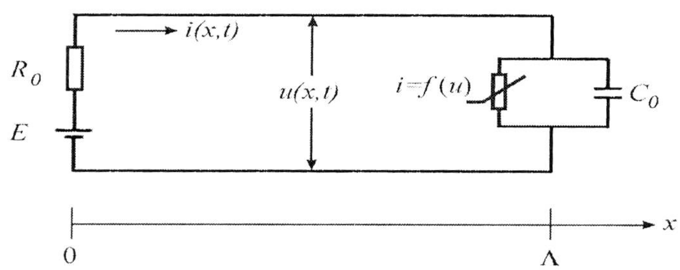
1. Introduction

A lot of papers have been devoted to the investigation of lossless transmission lines terminated by nonlinear loads (cf. [1], [16], [18], [4], [5], [12], [6], [9], [7], [3], [19]) and their applications to RF-circuits. It is well known, however, that in many practical cases (in particular when the frequency increases) the lossies can not be disregarded (cf. [9], [7]).That is way, here we consider a hyperbolic system describing the processes in the transmission lines taking into account the lossies, that is, $R \neq 0$ and $G \neq 0$ (cf. system (1) below). On the other hand in many devices [14] the V-I characteristics of nonlinear R-loads are approximated most often by polynomials. This implies arising of polynomial nonlinearities in the equivalent neutral functional differential equation (cf. [6]).

The main purpose of the present paper is twofold: to expose known results in a form (in Section II) for the direct application to analysis and design of lossy transmission lines and to present some new results for the existence of periodic regimes of lossy transmission lines without distortion terminated by nonlinear resistive loads (Section III). Finally in Section IV we show of how to apply the theorems proved to a concrete example - an important detail usually missing in the mathematical papers. We would like also to point out that our fixed point method (cf. [2]) combined with suitable Bielecki metrics overcomes the difficulties generated by polynomial nonlinearities. Unlike [19] our periodic solution can be approximated by combination of *sin* and *cos* in an explicit form.

We proceed from the circuit shown on the next figure where E is the source, R_0 and C_0 - linear loads, while the nonlinear resistive load has the nonlinear V-I characteristic of polynomial type.



It is known that a lossy transmission line can be prescribed by the following hyperbolic system of first order partial differential equations:

$$C\frac{\partial u(x,t)}{dt}+\frac{\partial i(x,t)}{dx}+Gu(x,t)=0, L\frac{\partial i(x,t)}{dt}+\frac{\partial u(x,t)}{dx}+Ri(x,t)=0,$$

$$(x,t)\in\Pi:=\left\{(x,t)\in\mathbb{R}^2:(x,t)\in[0,\Lambda]\times[0,\infty)\right\} \quad (1)$$

where $u(x,t)$ and $i(x,t)$ are the unknown voltage and current, while L,C, R and G are prescribed specific parameters of the line and $\Lambda>0$ is its length.

For the above system (1) can be formulated the following initial-boundary (or briefly mixed) problem: to find the unknown functions $u(x,t)$ and $i(x,t)$ in Π such that

$$u(x,0)=u_0(x), i(x,0)=i_0(x), x\in[0,\Lambda] \tag{2}$$

$$E-u(0,t)-R_0 i(0,t)=0, t\geq 0 \tag{3}$$

$$C_0\frac{du(\Lambda,t)}{dt}=i(\Lambda,t)-f(u(\Lambda,t)), t\geq 0. \tag{4}$$

where $i_0(x), u_0(x)$ are prescribed functions-the current and voltage at the initial instant, and $i=f(u)$ is a prescribed V-I characteristic of the nonlinear resistive load. For the sake of simplicity we assume that $f(u)$ is a polynomial of third order but the calculations below show that the method can be applied to polynomials of an arbitrary order. It is known that the third order polynomials are commonly used for the approximation in the oscillator circuits where $f(.)$ is a polynomial with partially negative differential resistance (cf. [9], [7], [14] Gunn-diodes, tunnel diodes and others).In the following we assume that the following condition is fulfilled $R/L=G/C$. It is a natural assumption for TEM way of propagation in the transmission lines (cf. [9], [7], [14]).

2. Reducing the mixed problem for the transmission line system to an initial value problem for a neutral equation

First we present (1) in matrix form:

$$A_1\frac{\partial U}{\partial t}+A_2\frac{\partial U}{\partial x}+A_3U=0 \tag{5}$$

where

$$A_1=\begin{bmatrix} C & 0\\ 0 & L\end{bmatrix},\qquad A_2=\begin{bmatrix} 0 & 1\\ 1 & 0\end{bmatrix},\qquad A_3=\begin{bmatrix} G & 0\\ 0 & R\end{bmatrix},$$

$$U=\begin{bmatrix} u\\ i\end{bmatrix},\qquad \frac{\partial U}{\partial t}=\begin{bmatrix} \frac{\partial u}{\partial t}\\ \frac{\partial i}{\partial t}\end{bmatrix}.$$

Since $|A_1|\neq 0$, then (5) becomes $\frac{\partial U}{\partial t}+A_1^{-1}A_2\frac{\partial U}{\partial x}+A_1^{-1}A_3U=0$, where

$$A_1^{-1}=\begin{bmatrix} 1/C & 0\\ 0 & 1/L\end{bmatrix}.$$

Therefore we obtain

$$\frac{\partial U}{\partial t} + A\frac{\partial U}{\partial x} + BU = 0, \tag{6}$$

where

$$A = \begin{bmatrix} 0 & 1/C \\ 1/L & 0 \end{bmatrix}, \quad B = \begin{bmatrix} G/C & 0 \\ 0 & R/L \end{bmatrix}.$$

In order to transform the matrix

$$A = A_1^{-1}A_2 = \begin{bmatrix} 0 & 1/C \\ 1/L & 0 \end{bmatrix}$$

in a diagonal form we have to solve the characteristic equation:

$$\begin{bmatrix} -\lambda & 1/C \\ 1/L & -\lambda \end{bmatrix} = 0$$

whose roots are $\lambda_1 = 1/\sqrt{LC}, \lambda_2 = -1/\sqrt{LC}$. For the eigen-vectors we obtain the following systems:

$$\left| \begin{array}{ll} -(1/\sqrt{LC})\xi_1 & +(1/L)\xi_2 = 0 \\ (1/C)\xi_1 & -(1/\sqrt{LC})\xi_2 = 0 \end{array} \right. \quad \text{and} \quad \left| \begin{array}{ll} (1/\sqrt{LC})\xi_1 & +(1/L)\xi_2 = 0 \\ (1/C)\xi_1 & +(1/\sqrt{LC})\xi_2 = 0 \end{array} \right.$$

Hence $(\xi_1^{(1)}, \xi_2^{(1)}) = (\sqrt{C}, \sqrt{L})$, $(\xi_1^{(2)}, \xi_2^{(2)}) = (-\sqrt{C}, \sqrt{L})$.

Denote by H the matrix formed by eigen-vectors $H = \begin{bmatrix} \sqrt{C} & \sqrt{L} \\ -\sqrt{C} & \sqrt{L} \end{bmatrix}$ and its inverse one $H^{-1} = \begin{bmatrix} 1/(2\sqrt{C}) & -1/(2\sqrt{C}) \\ 1/(2\sqrt{L}) & 1/(2\sqrt{L}) \end{bmatrix}$. If we denote by

$$A^{can} = \begin{bmatrix} 1/\sqrt{LC} & 0 \\ 0 & -1/\sqrt{LC} \end{bmatrix},$$

then it is known that $A^{can} = HAH^{-1}$.

Introduce new variables $Z = HU$, (or $U = H^{-1}Z$)

$$Z = \begin{bmatrix} V(x,t) \\ I(x,t) \end{bmatrix}, \qquad H = \begin{bmatrix} \sqrt{C} & \sqrt{L} \\ -\sqrt{C} & \sqrt{L} \end{bmatrix}, \qquad U = \begin{bmatrix} u(x,t) \\ i(x,t) \end{bmatrix}.$$

Then

$$\left| \begin{array}{l} V = \sqrt{C}u + \sqrt{L}i \\ I = -\sqrt{C}u + \sqrt{L}i \end{array} \right. , \qquad \left| \begin{array}{l} u = (1/2\sqrt{C})V - (1/2\sqrt{C})I \\ i = (1/2\sqrt{L})V + (1/2\sqrt{L})I \end{array} \right. \tag{7}$$

Replacing $U = H^{-1}Z$ in (6) we obtain

$$\frac{\partial(H^{-1}Z)}{\partial t} + A\frac{\partial(H^{-1}Z)}{\partial x} + B(H^{-1}Z) = 0.$$

Since H^{-1} is a constant matrix we have:

$$H^{-1}\frac{\partial Z}{\partial t} + (AH^{-1})\frac{\partial Z}{\partial x} + (BH^{-1})Z = 0.$$

After multiplication from the left by H we obtain

$$\frac{\partial Z}{\partial t} + H(AH^{-1})\frac{\partial Z}{\partial x} + H(BH^{-1})Z = 0,$$

i.e.

$$\frac{\partial Z}{\partial t} + A^{can}\frac{\partial Z}{\partial x} + H(BH^{-1})Z = 0. \tag{8}$$

But

$$HBH^{-1} = \begin{bmatrix} \sqrt{C} & \sqrt{L} \\ -\sqrt{C} & \sqrt{L} \end{bmatrix} \begin{bmatrix} G/C & 0 \\ 0 & R/L \end{bmatrix} \begin{bmatrix} 1/2\sqrt{C} & -1/2\sqrt{C} \\ 1/2\sqrt{L} & 1/2\sqrt{L} \end{bmatrix} =$$

$$= \begin{bmatrix} (1/2)(G/C + R/L) & (1/2)(-G/C + R/L) \\ (1/2)(-G/C + R/L) & (1/2)(G/C + R/L) \end{bmatrix}.$$

As we have already mentioned we consider transmission lines without distortion which means that the following condition is fulfilled $R/L = G/C$ (cf. [9], [7]). Then HBH^{-1} can be simplified and (8) has the type:

$$\begin{bmatrix} \frac{\partial V}{\partial t} \\ \frac{\partial I}{\partial t} \end{bmatrix} + \begin{bmatrix} \frac{1}{\sqrt{LC}} & 0 \\ 0 & -\frac{1}{\sqrt{LC}} \end{bmatrix} \begin{bmatrix} \frac{\partial V}{\partial x} \\ \frac{\partial I}{\partial x} \end{bmatrix} + \begin{bmatrix} \frac{R}{L} & 0 \\ 0 & \frac{R}{L} \end{bmatrix} \begin{bmatrix} V \\ I \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \tag{9}$$

The new initial conditions we obtain from (7) and (2):

$$\begin{aligned} V(x,0) &= \sqrt{C}u(x,0) + \sqrt{L}i(x,0) \\ &= \sqrt{C}u_0(x) + \sqrt{L}i_0(x) \equiv V_0(x), x \in [0,\Lambda], \end{aligned} \tag{10}$$

$$I(x,0) = -\sqrt{C}u(x,0) + \sqrt{L}i(x,0) = -\sqrt{C}u_0(x) + \sqrt{L}i_0(x) \equiv I_0(x), x \in [0,\Lambda].$$

Then the new boundary conditions become

$$\begin{aligned} E &- \left(\frac{1}{2\sqrt{C}}V(0,t) - \frac{1}{2\sqrt{C}}I(0,t)\right) \\ &-R_0\left(\frac{1}{2\sqrt{L}}V(0,t) + \frac{1}{2\sqrt{L}}I(0,t)\right) = 0 \end{aligned} \tag{11}$$

$$C_0\left[\frac{1}{2\sqrt{C}}\frac{dV(\Lambda,t)}{dt}-\frac{1}{2\sqrt{C}}\frac{dI(\Lambda,t)}{dt}\right]=\frac{1}{2\sqrt{L}}V(\Lambda,t)+\frac{1}{2\sqrt{L}}I(\Lambda,t)$$
$$-f\left(\frac{1}{2\sqrt{C}}V(\Lambda,t)- -\frac{1}{2\sqrt{C}}I(\Lambda,t)\right),t\geq 0.$$

So we have obtained the mixed problem (9)-(11) equivalent to (1)-(4). One can simplify (9) by the next substitution:

$$W(x,t)=e^{\frac{R}{L}t}V(x,t),J(x,t)=e^{\frac{R}{L}t}I(x,t),$$

i.e.

$$V(x,t)=e^{-\frac{R}{L}t}W(x,t),I(x,t)=e^{-\frac{R}{L}t}J(x,t) \tag{12}$$

System (9) can be written in the form:

$$\frac{\partial V}{\partial t}+\frac{1}{\sqrt{LC}}\frac{\partial V}{\partial x}+\frac{R}{L}V=0,\frac{\partial I}{\partial t}-\frac{1}{\sqrt{LC}}\frac{\partial I}{\partial x}+\frac{R}{L}I=0. \tag{13}$$

Then substituting $V(x,t)$ and $I(x,t)$ from (12) into (13) we obtain

$$\left|\ \frac{\partial W}{\partial t}+\frac{1}{\sqrt{LC}}\frac{\partial W}{\partial x}=0,\quad \frac{\partial J}{\partial t}-\frac{1}{\sqrt{LC}}\frac{\partial J}{\partial x}=0\ \right. . \tag{14}$$

System (14) correspondences to a lossless transmission line and then following [9] one can reduce the mixed problem for (14) to an equivalent (cf. [6]) initial value problem for a functional differential equation of neutral type ([15], [10], [13]) on the right boundary. The neutral equation is a nonlinear one in view of the nonlinear $V-I$ characteristic of the resistive load. In what follows we make this reduction.

Indeed, it is known [11] that the solution of (14) is a pair of functions $W(x,t)=\Phi_W(x-\omega t)$ and $J(x,t)=\Phi_J(x+\omega t)$ where Φ_W and Φ_J are arbitrary smooth functions while $\omega=1/\sqrt{LC}$ is the propagation velocity of waves. In view of (7) and (12) we obtain

$$u(x,t)=\frac{e^{-\frac{R}{L}t}}{2\sqrt{C}}[\Phi_W(x-\omega t)-\Phi_J(x+\omega t)],$$
$$i(x,t)=\frac{e^{-\frac{R}{L}t}}{2\sqrt{L}}[\Phi_W(x-\omega t)+\Phi_J(x+\omega t)]. \tag{15}$$

Hence

$$\Phi_W(x-\omega t)=e^{\frac{R}{L}t}\left(\sqrt{C}u(x,t)+\sqrt{L}i(x,t)\right),$$
$$\Phi_J(x+\omega t)=e^{\frac{R}{L}t}\left(\sqrt{L}i(x,t)+\sqrt{C}U(x,t)\right). \tag{16}$$

For $x = \Lambda$ we obtain

$$\Phi_W(\Lambda - \omega t) = e^{\frac{R}{L}t}\left[\sqrt{C}u(\Lambda, t) + \sqrt{L}i(\Lambda, t)\right],$$
$$\Phi_J(\Lambda + \omega t) = e^{\frac{R}{L}t}\left(\sqrt{L}i(\Lambda, t) + \sqrt{C}U(\Lambda, t)\right). \tag{17}$$

Let us put $\Lambda - \omega t = -\omega t' \Longrightarrow t = t' + \Lambda/\omega \equiv t' + T(T = \Lambda/\omega)$ and then substitute t in the first equation of (17) we get

$$\Phi_W(-\omega t') = e^{\frac{R}{L}(t'+T)}\left[\sqrt{C}u(\Lambda, t' + T) + \sqrt{L}i(\Lambda, t' + T)\right].$$

Now let us put $\Lambda + \omega t = \omega t'' \Longrightarrow t = t'' - \Lambda/\omega \equiv t'' - T(T = \Lambda/\omega)$ and then substitute in the second of equation of (17) one can obtain

$$\Phi_J(\omega t'') = e^{\frac{R}{L}(t''-T)}\left[\sqrt{L}i(\Lambda, t'' - T) - \sqrt{C}u(\Lambda, t'' - T)\right].$$

So we obtain

$$\Phi_W(-\omega t) = e^{\frac{R}{L}(t+T)}\left[\sqrt{C}u(\Lambda, t + T) + \sqrt{L}i(\Lambda, t + T)\right], \tag{18}$$
$$\Phi_J(\omega t) = e^{\frac{R}{L}(t-T)}\left[\sqrt{L}i(\Lambda, t - T) - \sqrt{C}u(\Lambda, t - T)\right]. \tag{19}$$

From (15) by $x = 0$ we have

$$u(0, t) = \frac{e^{-\frac{R}{L}t}}{2\sqrt{C}}[\Phi_W(-\omega t) - \Phi_J(\omega t)], i(0, t) = \frac{e^{-\frac{R}{L}t}}{2\sqrt{L}}[\Phi_W(-\omega t) + \Phi_J(\omega t)]. \tag{20}$$

Substituting $\Phi_W(-\omega t)$ and $\Phi_J(\omega t)$ from (18) and (19) into (20) we have:

$$u(0, t) = \frac{1}{2\sqrt{C}}\left[e^{\frac{RT}{L}}\left(\sqrt{C}u(\Lambda, t + T) + \sqrt{L}i(\Lambda, t + T)\right) - \right.$$
$$\left. - e^{-\frac{RT}{L}}\left(\sqrt{L}i(\Lambda, t - T) + \sqrt{C}u(\Lambda, t - T)\right)\right],$$
$$i(0, t) = \frac{1}{2\sqrt{L}}\left[e^{\frac{RT}{L}}\left(\sqrt{C}u(\Lambda, t + T) + \sqrt{L}i(\Lambda, t + T)\right) - \right.$$
$$\left. - e^{-\frac{RT}{L}}\left(\sqrt{L}i(\Lambda, t - T) + \sqrt{C}u(\Lambda, t - T)\right)\right].$$

Now we substitute the above expressions in the boundary condition $E - u(0, t) - R_0 i(0, t) = 0$ and using the usually accepted denotation for the characteristic impedance $Z_0 = \sqrt{L/C}$we have:

$$2E - e^{\frac{R\Lambda}{Z_0}}\left(1 + \frac{R_0}{Z_0}\right)u(\Lambda, t + T) - e^{\frac{R\Lambda}{Z_0}}(Z_0 + R_0)i(\Lambda, t + T) +$$
$$+ e^{-\frac{R\Lambda}{Z_0}}(Z_0 - R_0)i(\Lambda, t - T) + e^{-\frac{R\Lambda}{Z_0}}\left(\frac{R_0}{Z_0} - 1\right)u(\Lambda, t - T) = 0.$$

Put $t' = t + T$. In view of $t - T = t' - 2T$ (and again replace t' by t) we obtain:

$$2E - e^{\frac{R\Lambda}{Z_0}}\left(1 + \frac{R_0}{Z_0}\right)u(\Lambda, t) - e^{\frac{R\Lambda}{Z_0}}(Z_0 + R_0)i(\Lambda, t) +$$

$$+e^{-\frac{R\Lambda}{Z_0}}(Z_0 - R_0)i(\Lambda, t - 2T) + e^{-\frac{R\Lambda}{Z_0}}\left(\frac{R_0}{Z_0} - 1\right)u(\Lambda, t - 2T) = 0. \qquad (21)$$

The second boundary condition is $C_0\dfrac{du(\Lambda, t)}{dt} = i(\Lambda, t) - f(u(\Lambda, t))$ or

$$i(\Lambda, t) = C_0\frac{du(\Lambda, t)}{dt} + f(u(\Lambda, t)) \qquad (22)$$

and

$$i(\Lambda, t - 2T) = C_0\frac{du(\Lambda, t - 2T)}{dt} + f(u(\Lambda, t - 2T)). \qquad (23)$$

Substituting $i(\Lambda, t)$ and $i(\Lambda, t - 2T)$ from (22) and (23) into (21) we obtain:

$$2E - e^{\frac{R\Lambda}{Z_0}}\left(1 + \frac{R_0}{Z_0}\right) - e^{\frac{R\Lambda}{Z_0}}(Z_0 + R_0)[C_0\dot{u}(\Lambda, t) + f(u(\Lambda, t))] +$$

$$e^{-\frac{R\Lambda}{Z_0}}(Z_0 + R_0)[C_0\dot{u}(\Lambda, t - 2T) + f(u(\Lambda, t - 2T))] +$$

$$+e^{-\frac{R\Lambda}{Z_0}}\left(\frac{R_0}{Z_0} - 1\right)u(\Lambda, t - 2T) = 0\left(\dot{u} \equiv \frac{du}{dt}\right).$$

Denote by $y(t) = u(\Lambda, t)$ and $A = e^{\frac{R\Lambda}{Z_0}}$ and solve the above equation with respect to $\dot{y}(t)$:

$$\dot{y}(t) = \frac{2E}{AC_0(Z_0 + R_0)} - \frac{1}{Z_0C_0}y(t) - \frac{1}{C_0}f(y(t)) + \frac{R_0 - Z_0}{A^2C_0Z_0(R_0 + Z_0)}y(t - 2T) +$$

$$+\frac{Z_0 - R_0}{A^2C_0(Z_0 + R_0)}f(y(t - 2T)) + \frac{Z_0 - R_0}{A^2(Z_0 + R_0)}\dot{y}(t - 2T). \qquad (24)$$

This is a nonlinear functional differential equation of neutral type (cf. [15], [10], [13]) with respect to the unknown function $y(t)$.

3. An existence of periodic solutions for neutral equations with polynomial nonlinearities

Our method for solving the periodic boundary value problem for the above equation is applicable to the general case when $V-I$ characteristic of the nonlinear resistive load is a polynomial of an arbitrary order, that is, when $f(u)=\sum_{k=1}^{n} a_k u^k$. We however consider the particular case very often encountered in the applications$f(u)=a_1u+a_2u^2+a_3u^3$. Then the above equation becomes

$$\dot{y}(t)=\frac{2E}{AC_0(Z_0+R_0)}-\frac{1}{Z_0C_0}y(t)-\frac{1}{C_0}(a_1y(t)+a_2y^2(t)+a_3y^3(t))$$

$$+\frac{R_0-Z_0}{A^2C_0Z_0(R_0+Z_0)}y(t-2T)++\frac{Z_0-R_0}{A^2C_0(Z_0+R_0)}(a_1y(t-2T)+a_2y^2(t-2T)$$

$$+a_3y^3(t-2T))+\frac{Z_0-R_0}{A^2(Z_0+R_0)}\dot{y}(t-2T).$$

or

$$\dot{y}(t)=\frac{2E}{AC_0(Z_0+R_0)}-\frac{1+a_1Z_0}{Z_0C_0}y(t)+\frac{a_2}{C_0}y^2(t)$$

$$+\frac{a_3}{C_0}y^3(t)+\frac{(1-a_1Z_0)(R_0-Z_0)}{A^2C_0Z_0(R_0+Z_0)}y(t-2T)++\frac{a_2(Z_0-R_0)}{A^2C_0(Z_0+R_0)}y^2(t-2T)$$

$$+\frac{a_3(Z_0-R_0)}{A^2C_0(Z_0+R_0)}y^3(t-2T)+\frac{Z_0-R_0}{A^2(Z_0+R_0)}\dot{y}(t-2T). \qquad (25)$$

Introduce the denotations:

$$A=e^{\frac{R\Lambda}{Z_0}},\quad A_0=\frac{2E}{AC_0(Z_0+R_0)},\quad A_1=-\frac{1+a_1Z_0}{Z_0C_0},\quad A_2=-\frac{a_2}{C_0},$$

$$A_3=-\frac{a_3}{C_0},\quad A_4=\frac{(1-a_1Z_0)(R_0-Z_0)}{A^2C_0Z_0(R_0+Z_0)},\quad A_5=\frac{a_2(Z_0-R_0)}{A^2C_0(Z_0+R_0)},$$

$$A_6=\frac{a_3(Z_0-R_0)}{A^2C_0(Z_0+R_0)},\quad A_7=\frac{Z_0-R_0}{A^2(Z_0+R_0)}.$$

Then for (25) one can formulate the following initial value problem: to find a solution of (25) on a prescribed interval $[T,T_0]$ where on the interval $[-T,T]$ the solution equals to a prescribed initial function, that is,$y(t)=\phi(t)$. As we show below $\phi(t)$ can be defined by the initial voltage $u_0(x)$. Indeed, in view of the characteristic lines of the hyperbolic system (6)$\eta-\omega t=x,\eta+\omega t=x$, we can translate the initial function from $[0,\Lambda]$ onto $[0,T]$ and from $[0,\Lambda]$ onto $[-T,0]$. So we obtain for the initial function

$$\phi(t)=\begin{cases} u_0(\Lambda+\omega t), t\in[-T,0] \\ u_0(\Lambda+-\omega t), t\in[0,T] \end{cases}$$

Therefore we have the following problem for the existence of To-periodic solution of the neutral equation:

$$\dot{y}(t)=A_0+A_1y(t)+A_2y^2(t)+A_4y(t-2T)+A_5y^2(t-2T)$$
$$+A_6y^3(t-2T)+A_7\dot{(}t-2T),\ \text{for } t\in[T,T_0]$$
$$y(t)=\phi(t),\dot{y}(t)=\dot{\phi}(t) \text{ for } t\in[-T,T]. \tag{26}$$

We have to present (26) in a suitable operator form. Prior to define the suitable operator we consider the set $C_{T_0}(\phi)$ of all continuous T_0-periodic functions on $[T,\infty)$ which coincide with $\phi(t)$ on $[-T,T]$. If we look for continuously differentiable solutions one can assume that the following compatibility condition is fulfilled:

$$(CC)\quad \dot{\phi}(T)=A_0+A_1\phi(T)+A_2\phi^2(T)+A_3\phi^3(T)+A_4\phi(-T)$$
$$+A_5\phi^2(-T)+A_6\phi^3(-T)+A_7\dot{\phi}(-T). \tag{27}$$

The above condition implies an existence of continuous derivative $\dot{y}(t)$ of the solution (cf. [15], [10], [13]).

Now introduce the set:

$$M=\left\{f(.)\in C_{T_0}(\phi):|f(t)|\le Ye^{\mu t},t\in[-T,T+T_0]\right\},$$

where the positive constants Y, T_0 and μ will be chosen below. It is easy to verify that M turns out into a complete metric space with respect to the metric $\rho_\mu(f,g)=sup\left\{e^{-\mu t}|f(t)-g(t)|:t\in[-T,T+T_0]\right\}<\infty$.

Now we are able to formulate the main result:

Theorem 1. Let the following conditions be fulfilled:

1.1) the initial function $\phi(t)$ satisfies condition (CC) and $\phi(-T)=0$;

1.2) the constants Y,T_0,μ are chosen such that $\mu T_0<2/3$ and the following inequalities are valid

$$|\phi(T)|+3T_0\frac{|A_0|e^{\mu T}}{2-\mu T_0}+KY\le Y$$

where $A = max\{|A_k| : k = 1, 2, ..., 7\}$ and

$$K = 3T_0 A\left[\frac{2ch(\mu T)}{2-\mu T_0} + \frac{4ch(2\mu T)}{2-2\mu T_0}Y + \frac{6ch(3\mu T)}{2-3\mu T_0}Y^2 + \frac{e^{\mu(T_0-T)}}{2}\right] < 1$$

Then there exists a unique T_0-periodic solution of the equation (26).

Proof. Denote by $F(f(t))$ the right-hand side of (26):

$$F(f(t)) = A_0 + A_1 f(t) + A_2 f^2(t) + A_3 f^3(t) + A_4 f(t-2T)$$
$$+A_5 f^2(t-2T) + A_6 f^3(t-2T) + A_7 \dot{f}(t-2T).$$

Define the operator $B : M \longrightarrow M$ by the formula (recall that $y(T) = \phi(T)$):

$$(Bf)(t) := \phi(T) + \int_T^t F(f(s))ds$$
$$-\left(\frac{t-T}{T_0} - \frac{1}{2}\right)\int_T^{T+T_0} F(f(s))ds \text{ for } t \in [T, \infty],$$
$$(Bf)(t) := \phi(T) \text{ and } d(Bf)(t)/dt = \dot{\phi}(t) \text{ for } t \in [-T, T]. \tag{28}$$

One can show that the existence of a continuous T_0-periodic solution of (26) is equivalent to the existence of a fixed point of the operator B in the set M (cf. [17]).

First we show that the operator B maps the set M into itself. Indeed the function $Bf)(t)$ is T_0-periodic on $[T, \infty]$:

$$(Bf)(t+T_0) := \phi(T) + \int_T^t F(f(s))ds + \int_t^{t+T_0} F(f(s))ds$$
$$-\left(\frac{t-T}{T_0} - \frac{1}{2}\right)\int_T^{T+T_0} F(f(s))ds - \frac{T_0}{T_0}\int_T^{T+T_0} F(f(s))ds$$
$$= \phi(T) + \int_T^t F(f(s))ds - \left(\frac{t-T}{T_0} - \frac{1}{2}\right)\int_T^{T+T_0} F(f(s))ds = (Bf)(t).$$

In what follows we have to show also that $|(Bf)(t)| \le Ye^{\mu t}$.

Indeed, in view of the inequalities

$$\left|\frac{t-T}{T_0} - \frac{1}{2}\right| \le \frac{1}{2}$$

and

$$\frac{e^h - 1}{h} \le \frac{h(1 + h/2 + (h/2)^2 + ...)}{h} \le \frac{1}{1-(h/2)} = \frac{2}{2-h}$$

we obtain:

$$
\begin{aligned}
(Bf)(t) &\le |\phi(T)| + \int_T^t |F(f(s))|ds + \frac{1}{2}\int_T^{T+T_0} |F(f(s))|ds \le |\phi(T)| \\
&+\frac{3}{2}\int_T^{T+T_0} |F(f(s))|ds \le \frac{3}{2}\int_T^{T+T_0} \Big[|A_0|e^{\mu s} + |A_1||f(s)|e^{\mu s}e^{-\mu s} \\
&+|A_2||f(s)|^2 e^{2\mu s}e^{-2\mu s} + |A_3|e^{3\mu s}e^{-3\mu s} \\
&+|A_4||f(s-2T)|e^{-\mu(s-2T)}e^{\mu(s-2T)} + |A_5||f(s-2T)|^2 e^{-2\mu(s-2T)}e^{2\mu(s-2T)} \\
&+|A_6||f(s-2T)|^3 e^{-3\mu(s-2T)}e^{3\mu(s-2T)}\Big] ds \\
&+\frac{3}{2}\Big[|A_7||f(T_0-T)|e^{-\mu(T_0-T)}e^{\mu(T_0-T)}\Big] \\
&\le |\phi(T)|e^{\mu t} + \frac{3}{2}\Bigg[|A_0|\frac{e^{\mu(T+T_0} - e^{\mu T}}{\mu} + |A_1|Y\frac{e^{\mu(T+T_0} - e^{\mu T}}{\mu} \\
&+|A_2|Y^2\frac{e^{2\mu(T+T_0} - e^{2\mu T}}{2\mu} + |A_3|Y^3\frac{e^{3\mu(T+T_0} - e^{3\mu T}}{3\mu} \\
&+|A_4|Ye^{-2\mu T}\frac{e^{\mu(T+T_0} - e^{\mu T}}{\mu} + +|A_5|Y^2 e^{-4\mu T}\frac{e^{2\mu(T+T_0} - e^{2\mu T}}{2\mu} \\
&+|A_6|Y^3 e^{-6\mu T}\frac{e^{3\mu(T+T_0} - e^{3\mu T}}{3\mu} + |A_7|Ye^{\mu(T_0-T)}\Bigg] \\
&\le |\phi(T)|e^{\mu t} + \frac{3}{2}\Bigg[|A_0|\frac{e^{\mu T}T_0(e^{\mu T_0}-1)}{\mu T_0} + |A_1|Y\frac{e^{\mu T}T_0(e^{\mu T_0}-1)}{\mu T_0} \\
&+|A_2|Y^2\frac{e^{2\mu T}T_0(e^{2\mu T_0}-1)}{2\mu T_0} + |A_3|Y^3\frac{e^{3\mu T}T_0(e^{3\mu T_0}-1)}{3\mu T_0} \\
&+|A_4|Ye^{-\mu T}\frac{T_0(e^{\mu T_0}-1)}{\mu T_0} + |A_5|Y^2 e^{-2\mu T}\frac{T_0(e^{2\mu T_0}-1)}{2\mu T_0} \\
&+|A_6|Y^3 e^{-3\mu T}\frac{T_0(e^{3\mu T_0}-1)}{3\mu T_0}\Bigg] + \frac{3}{2}|A_7|Ye^{\mu(T_0-T)} \\
&\le e^{\mu t}|\phi(T)| + e^{\mu t}\frac{3T_0}{2}\Bigg[\frac{2(|A_0|+|A_1|Y)e^{\mu T}}{2-\mu T_0} + \frac{2|A_2|Y^2 e^{2\mu T}}{2-2\mu T_0} + \frac{2|A_3|Y^3 e^{3\mu T}}{2-3\mu T_0} \\
&+\frac{2|A_4|Ye^{-\mu T}}{2-\mu T_0} + \frac{2|A_5|Y^2 e^{-2\mu T}}{2-2\mu T_0} + \frac{2|A_6|Y^3 e^{-3\mu T}}{2-3\mu T_0}\Bigg] + e^{\mu t}\frac{3}{2}|A_7|Ye^{\mu(T_0-T)} \\
&\le e^{\mu t}|\phi(T)| + e^{\mu t}3T_0\Bigg[\frac{|A_0|e^{\mu T}}{2-\mu T_0} + \frac{|A_1|e^{\mu T} + |A_4|e^{-\mu T}}{2-\mu T_0}Y \\
&+\frac{|A_2|e^{2\mu T} + |A_5|e^{-2\mu T}}{2-2\mu T_0}Y^2
\end{aligned}
$$

$$+\frac{|A_3|e^{3\mu T}+|A_6|e^{-3\mu T}}{2-3\mu T_0}Y^3\Big]+e^{\mu t}\frac{3}{2}|A_7|Ye^{\mu(T_0-T)}\le e^{\mu t}Y.$$

The last inequality is satisfied in view of condition 1.2) of Theorem 1.

Consequently the operator B maps M into itself.

It remains to show that B is a contractive operator. Indeed for every two functions $f(.)$ and $g(.)$ from M we have:

$$|(Bf)(t)-(Bg)(t)|\le\int_T^t\big[|A_1||f(s)-g(s)|e^{-\mu s}e^{\mu s}+|A_2||f^2(s)-g^2(s)|$$
$$+|A_3||f^3(s)-g^3(s)|+|A_4||f(s-2T)-g(s-2T)|e^{-\mu(s-2T)}e^{\mu(s-2T)}$$
$$+|A_5||f^2(s-2T)-g^2(s-2T)|+|A_6||f^3(s-2T)-g^3(s-2T)|\big]\,ds$$
$$+|A_7||f(t-2T)-g(t-2T)|e^{-\mu(t-2T)}e^{\mu(t-2T)}$$
$$+\int_T^{T+T_0}\big[|A_1||f(s)-g(s)|e^{-\mu s}e^{\mu s}$$
$$+|A_2||f^2(s)-g^2(s)|+|A_3||f^3(s)-g^3(s)|$$
$$+|A_4||f(s-2T)-g(s-2T)|e^{-\mu(s-2T)}e^{\mu(s-2T)}$$
$$+|A_5||f^2(s-2T)-g^2(s-2T)|+|A_6||f^3(s-2T)-g^3(s-2T)|\big]\,ds$$
$$+|A_7||f(T_0-T)-g(T_0-T)|e^{-\mu(T_0-T)}e^{\mu(T_0-T)}$$
$$\le\rho_\mu(f,g)\frac{3}{2}\left[|A_1|\frac{e^{\mu(T+T_0)}-e^{\mu T}}{\mu}+2|A_2|Y\int_T^{T+T_0}e^{2\mu s}ds\right.$$
$$+3|A_3|Y^2\int_T^{T+T_0}e^{3\mu s}ds++[|A_4|e^{-2\mu T}\frac{e^{\mu(T+T_0)}-e^{\mu T}}{\mu}$$
$$+2|A_5|Y\int_T^{T+T_0}e^{2\mu(s-2T)}ds+3|A_6|Y^2\int_T^{T+T_0}e^{3\mu(s-2T)}ds$$
$$\left.+|A_7|e^{\mu(T_0-T)}\right]\le\rho_\mu(f,g)\frac{3}{2}\left[|A_1|\frac{e^{\mu T}(e^{\mu T_0}-1)}{\mu}+2|A_2|Y\frac{e^{2\mu(T+T_0)}-e^{2\mu T}}{2\mu}\right.$$
$$+3|A_3|Y^2\frac{e^{3\mu(T+T_0)}-e^{2\mu T}}{3\mu}+|A_4|e^{-2\mu T}\frac{e^{\mu(T+T_0)}-e^{\mu T}}{\mu}$$
$$+2|A_5|Ye^{-4\mu T}\frac{e^{2\mu(T+T_0)}-e^{2\mu T}}{2\mu}$$
$$\left.+3|A_6|Y^2e^{-6\mu T}\frac{e^{3\mu(T+T_0)}-e^{3\mu T}}{3\mu}+|A_7|e^{\mu(T_0-T)}\right]$$
$$\le\rho_\mu(f,g)\frac{3}{2}\left[|A_1|e^{\mu T}T_0\frac{e^{\mu T_0}-1}{\mu T_0}\right.$$

$$+2|A_2|YT_0e^{2\mu T}\frac{e^{2\mu T_0}-1}{2\mu T_0}+3|A_3|Y^2T_0e^{3\mu T}\frac{e^{3\mu T_0}-1}{3\mu T_0}+|A_4|e^{-\mu T}T_0\frac{e^{\mu T_0}-1}{\mu}$$

$$+2|A_5|Ye^{-2\mu T}T_0\frac{e^{2\mu T_0}-1}{2\mu T_0}+3|A_6|Y^2e^{-3\mu T}T_0\frac{e^{3\mu T_0}-1}{3\mu T_0}+|A_7|e^{\mu(T_0-T)}\Bigg]$$

$$\leq e^{\mu t}\rho_\mu(f,g)\frac{3T_0}{2}\left[\left(|A_1|e^{\mu T}+|A_4|e^{-\mu T}\right)\frac{2}{2-\mu T_0}\right.$$

$$+2\left(|A_2|e^{2\mu T}+|A_5|e^{-2\mu T}\right)\frac{2}{2-2\mu T_0}Y$$

$$\left.+3\left(|A_3|e^{3\mu T}+|A_6|e^{-3\mu T}\right)\frac{2}{2-3\mu T_0}Y^2+|A_7|e^{\mu(T_0-T)}\right]$$

$$\leq e^{\mu t}\rho_\mu(f,g)3T_0\left[\frac{|A_1|e^{\mu T}+|A_4|e^{-\mu T}}{2-\mu T_0}+\frac{2\left(|A_2|e^{2\mu T}+|A_5|e^{-2\mu T}\right)}{2-2\mu T_0}Y\right.$$

$$\left.+\frac{3\left(|A_3|e^{3\mu T}+|A_6|e^{-3\mu T}\right)}{2-3\mu T_0}Y^2+\frac{|A_7|}{2}e^{\mu(T_0-T)}\right]\leq e^{\mu t}K\rho_\mu(f,g).$$

Now we multiply the both sides of the above inequality by $e^{-\mu t}$ and taking the supremum we obtain $\rho_\mu(Bf,Bg)\leq K\rho_\mu(f,g)$ .Consequently the operator B is contractive one in view of condition 1.3) because $K<1$. The contraction mapping principle implies an existence of unique T_0-periodic solution of (26). Theorem 1 is thus proved.

The approximated solution we obtain in the concrete example below.

Very often compatibility condition (CC) is restrictive. We can omit it by considering a different function space. Indeed, we can look for a solution in the space in T_0-periodic absolutely continuous functions with derivatives belonging to the space $L^\infty[-T,\infty)$ -measurable functions with essentially bounded derivatives (cf. [8]). Then the set M can be defined as follows:

$$M=\left\{f(.)\in C_{T_0}(\phi):|f(t)|\leq Ye^{\mu t}\ \text{ for almost all }\ t\in[-T,T+T_0]\right\}.$$

The metric could be defined as follows:

$$\rho_\mu(f,g)=ess\sup\left\{e^{-\mu t}|f(t)-g(t)|:t\in[-T,T+T_0]\right\}<\infty$$

Then the following theorem is valid:

Theorem 2. Let the conditions 1.2) and 1.3) be fulfilled. Then there exists a unique absolutely continuous T_0-periodic solution of the equation (26) whose derivative belongs to $L^\infty[-T,\infty)$.

The proof is analogous to the one of the previous theorem.

The next theorem treats the particular case when $\mu = 0$. Then $M = \{f(.) \in C_{T_0}(\phi) : |f(t)| \leq Y$ for almost all $t \in [-T, T+T_0]\}$. The metric could be defined as follows:

$$\rho_\mu(f,g) = ess \sup \{|f(t) - g(t)| : t \in [-T, T+T_0]\} < \infty.$$

Theorem 3. Let the following conditions be fulfilled:
3.1 $\phi(T) = \phi(-T) = 0$; 3.2 $2T_0|A_0| + YK \leq Y$; 3.3 $K = 2T_0 A(4 + 3Y + 6Y^2) < 1$
for suitably chosen T_0, Y.

Then there exists a unique absolutely continuous T_0-periodic solution of the equation (26) whose derivative belongs to $L^\infty[-T, \infty)$.

The proof is analogous to the one of the previous theorems.

4. Conclusion-an example

Finally we show of how to apply the above Theorem 3 to the analysis and design of a lossy transmission line. We consider the above problem, when the nonlinear resistive load has a third order polynomial $V - I$ characteristic. Namely, the mixed problem corresponding to Fig.1 is:

$$C\frac{\partial u(x,t)}{\partial t} + \frac{\partial i(x,t)}{\partial x} + Gu(x,t) = 0, L\frac{\partial i(x,t)}{\partial t} + \frac{\partial u(x,t)}{\partial x} + Ri(x,t) = 0,$$
$$(x,t) \in \Pi = \left\{(x,t) \in \Pi^2 : (x,t) \in [0,\Lambda] \times [0,\infty)\right\},$$
$$u(x,0) = u_0(x), i(x,0) = i_0(x), x \in [0,\Lambda],$$
$$E - u(0,t) - R_0 i(0,t) = 0, t \geq 0,$$
$$C_0 \frac{du(\Lambda,t)}{dt} = i(\Lambda,t) - a_1 u(\Lambda,t) - a_2 u(\Lambda,t) - a_3 u^3(\Lambda,t), t \geq 0$$

can be reduced to the periodic problem for the following neutral functional differential equation with unknown function $y(t) = u(\Lambda, t)$:

$$\dot{y}(t) = \frac{2E}{AC_0(Z_0+R_0)} - \frac{1+a_1Z_0}{Z_0C_0}y(t) - \frac{a_2}{C_0}\frac{a_3}{C_0}y^3(t)$$
$$+\frac{(1-a_1Z_0)(R_0-Z_0)}{A^2C_0Z_0(R_0+Z_0)}y(t-2T) + \frac{a_2(Z_0-R_0)}{A^2C_0(Z_0+R_0)}y^2(t-2T)$$
$$+\frac{a_3(Z_0-R_0)}{A^2C_0(Z_0+R_0)}y^3(t-2T) + \frac{Z_0-R_0}{A^2(Z_0+R_0)}\dot{y}(t-2T), t \in [T, T+T_0], \quad (29)$$
$$y(t) = \phi(t), t \in [-T,T], \dot{y}(t) = \dot{\phi}(t), t \in [-T,T],$$

where the initial function is

$$\phi(t) = \begin{cases} u_0(\Lambda + \omega t), t \in [-T, 0] \\ u_0(\Lambda + -\omega t), t \in [0, T] \end{cases}$$

Let us consider a transmission line with length $\Lambda = 9cm$, cross-section area $S = 0,10cm^2$, specific resistance for the cuprum is $\rho_c = 0,0175$, characteristic impedance is $Z_0 = \sqrt{L/C} = 75\Omega, R_0 = 70\Omega, C_0 = 10^{-11}F$, $E = 0,5V$. The propagation velocity for the air is $c = 3.10^8 cm/sec$. This means $\omega = 1/\sqrt{L/C} = 3.10^{10}$. Then $T = \Lambda/\omega = \dfrac{9}{3.10^{10}} = 3.10^{-10}$. We use the following $V - I$ characteristic $i = 0,028u - 0,125u^2 + 0,14u^3$,i.e. $a_1 = 0,028, a_2 = -0,125, a_3 = 0,14$. Then the values of the constants are

$$A = e^{\frac{RA}{Z_0}} = e^{\rho \frac{\Lambda^2}{SZ_0}} = e^{1,75.10^{-2} \frac{9^2}{0,1.75}} = e^{0,189} \approx 1,224;$$

$$A^2 = 1,5;$$

$$A_0 = \frac{1}{1,224.10^{-11}.145} \approx 5,6.10^8;$$

$$A_1 = -\frac{1 + 0,028.75}{75.10^{-11}} \approx -4,1.10^9;$$

$$A_2 = -\frac{-0,125}{10^{-11}} \approx 1,25.10^{10};$$

$$A_3 = \frac{0,14}{10^{-11}} \approx 1,4.10^{10};$$

$$A_4 = \frac{(1 - 0,028.75)(-5)}{1,5.10^{-11}.75.145} \approx 3,3.10^7$$

$$A_5 = \frac{-0,01.5}{1,5.10^{-11}.75.145} \approx -3.10^5;$$

$$A_6 = \frac{0,14.5}{1,5.10^{-11}.145} \approx 3,2.10^8;$$

$$A_7 = \frac{5}{1,5.145} \approx 0,023.$$

One can choose $T_0 = 10^{-12}$. The conditions of Theorem 3 become:
$A = 1,4.10^{10}, K = 2,8.10^{-2}(4 + 3Y + 6Y^2) < 1, \quad 1,12.10^{-3} + YK \le Y.$
Consequently Y can be defined from the last inequalities.

In what follows we show of how to obtain several successive approximations of the solution. We proceed from (27)-(28). Let us choose

$$y_0(t) = \begin{cases} sin\left(\frac{2\pi}{T}\right) t, t \in [-T, T] \\ 0, \quad t \in [T, T + T_0] \end{cases}$$

Then substituting in the right hand-side of (29) we obtain:

$$y_1(t) = A_7 sin\frac{2\pi t}{T} + \left(\frac{t-T}{T_0} - \frac{1}{2}\right)\frac{A_7}{2} sin\frac{2\pi T_0}{T}.$$

Since $t \in [T, T+T_0]$ then

$$|y_1(t) - y_0(t)| \leq \frac{5}{4}|A_7| \leq 1,25.0,023 = 0,02875.$$

It is known that if $y^*(t)$ is the solution then

$$|y_n(t) - y^*(t)| \leq \frac{K^n}{1-K} 0,03.$$

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