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WEAKLY PICARD OPERATORS: EQUIVALENT DEFINITIONS, APPLICATIONS AND OPEN PROBLEMS

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Abstract. The purpose of this paper is to present several characterizations for the concept of weakly Picard operator in K-metric spaces. Some new characterizations and applications, as well as, several open questions are also discussed.
Key Words and Phrases: Fixed point, L-space, (weakly) Picard operator, K-metric, Caristi operator, Schröder's pair, invariant subset, maximal element, progressive operator.
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