

## WHO AUTHORED THE FIRST INTEGRAL EQUATIONS BOOK IN THE WORLD?

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**Abstract.** Traian Lalescu (1882-1929), a great Romanian mathematician, authored the first book on integral equations.

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1. The theory of integral equations is an active field of mathematics. To give an idea of the theoretical and practical importance of this field we present the following remarks and considerations:

- Many problems arising in physics, chemistry, biology, astronomy, theoretical mechanics, elasticity, plasticity, engineering, economics, medicine (etc.) lead to mathematical models described by integral equations ([9], [10], [13], [14], [15], [18], [23], [30], [44])
- Many problems of ordinary differential and partial differential equations can be studied more efficiently by integral equation methods ([8],[9], [10], [13], [14], [18], [25], [26], [40]-[46])
- Exact solutions of integral equations play an important role in ([18]):
  - understanding and to study of problems of natural sciences
  - to verify the consistency and estimate errors of various numerical and approximate methods
- The theory of integral equations is based on Real and Complex analysis, Fourier analysis, Functional analysis, Operator theory, Topology and Numerical analysis ([16], [19], [22], [24], [25], [27], [33], [34], [37], [38])
- The main subfields of the Integral equations are:
  - Fredholm integral equations
  - Volterra integral equations

- Linear integral equations
  - Nonlinear integral equations
  - Singular integral equations
  - Integro-ordinary differential equations
  - Integro-partial differential equations
  - Integral equations in abstract spaces
  - Abstract integral equations
  - Random integral equations
  - Eigenvalue problems
  - Qualitative behavior
  - Theoretical approximation of solutions
  - Numerical approximation of solutions
  - Exact solutions
  - Inverse problems
- There exists journals devoted to the publication of current research in Integral equation:
    - Integral equations and operator theory
    - Journal of integral equations
    - Journal of integral equations and applications

**2. Traian Lalescu** (1882-1929) is a great Romanian mathematician ([8], [15], [39]). He has results in many fields of mathematics. The main Lalescu's results are in the theory of integral equations. He authored the first **book** on integral equations ([3a], [3b], [3c], [3d], [1a], [1b], [2], [39], [6], [9], [11], [13]). His Ph.D. dissertation is "Sur l'équation de Volterra" (Paris, 1908). He has deep papers on this field. Here are some of them:

- Sur l'ordre de la fonction  $D(\lambda)$  de Fredholm, C.R. Acad. Sci., 145(1907), 906-907.
- Sur la fonction  $D(\lambda)$  de Fredholm, C.R. Acad. Sc., 145(1907), 1136-1139.
- L'étude des noyaux résolvants, Bull. de la Soc., Math. de France, 39(1911), 85-103.
- Sur l'application des équations intégrales à la théorie des équations différentielles, C.R. Acad. Sc., 166(1918), 727-728.
- Quelques théorèmes sur les noyaux symétrisables, Bull. des Sc. Math. (București), 28(1925), 25-30.

The bibliographical investigations make it evident that Lalescu's book was of the great importance ([6], [13], [11], [15]). V. Volterra (see *Revista Universitară Matematică* (București), 1(1929)), for example, has the following appreciation: "Introduction à la théorie des équations intégrales" e un' opera eccellente d'insime su ciò che era stato fatto sino a quel momento sulla teoria delle equazioni integrali."

3. After ninety years from the Lalescu's book many mathematicians authored books on integral equations. Here are some of them: M. Bocher (1912), D. Hilbert (1912), V. Volterra (1913), T. Carleman (1923), E. Goursat (1923), Hellinger-Toeplitz (1928), Vivanti-Schwank (1929), G. Viarda (1933), H.M. Müntz (1934), I.S. Privalov (1934), I.I. Privalov (1935), N.I. Muskhelishvili (1946, 1953, 1992), W.V. Lovitt (1950), N.S. Smirnov (1951), Courant-Hilbert (1953), F.G. Tricomi (1957, 1960, 1985), I.G. Petrovskii (1957, 1965), S.G. Mikhailin (1957, 1959), F. Smithies (1958), M.A. Krasnoselskii (1964), F.D. Gakhov (1966), W. Pogorzelskii (1966), L.G. Mikhailov (1966), G. Hoheisel (1967), Krasov-Kiselev-Macarenco (1968), C.D. Green (1969), H. Widam (1969), J.A. Cochran (1972), H. Hochstadt (1973), C. Corduneanu (1973, 1991), Chambers (1976), C.T.H. Baker (1977), Jaswon-Symm (1977), B.L. Moiseiwitsch (1977), J. Král (1980), K. Jörgens (1982), Colton-Kress (1983), Delves-Mohamed (1985), A.J. Jerry (1985), Brunner-Houwen (1986), A.D. Dzuraev (1987), R. Kress (1989), Gorenflo-Vessela (1991), J. Kondo (1991), A.C. Pipkin (1991), Porter-Stirling (1990), R. Precup (1993), R.P. Kanwal (1996), J.K. Lifanov (1996), Hackbusch (1995), L.A. Saknovich (1996), O'Regan-Meehan (1998), Polyanin-Manzhairov (1998), M. Văth (1999), Mandal-Mandal (1999), Appell-Kalitvin-Zabrejko (2000), Estrada-Kanwal (2000),...

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