ASYMPTOTIC EXPANSIONS FOR FAVARD OPERATORS

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In 1944, J. Favard introduced the operator F_n given by

$$(F_n f)(x) = \frac{1}{\sqrt{\pi n}} \sum_{\nu=-\infty}^{\infty} f\left(\frac{\nu}{n}\right) \exp\left(-n\left(\frac{\nu}{n} - x\right)^2\right)$$

which is a discrete analogue of the Gauss–Weierstrass singular convolution integral. The sequence $(F_n f)$ converges to f for continuous functions defined on $\mathbb{R}$ satisfying certain growth conditions. We report some known properties such as saturation theorems in certain polynomial weight spaces and present Kantorovich and Durrmeyer variants.

In fact, we treat a slight generalization F_{n,σ_n} which was introduced and studied by Gawronski and Stadtmüller. We study the local rate of convergence for smooth functions. The main result is a complete asymptotic expansion for the sequence $(F_{n,\sigma_n} f)$ as n tends to infinity. Furthermore, we consider a truncated version of these operators which possesses the same asymptotic properties. All results were proved also for simultaneous approximation.

Some recent results are joint work with Prof. Paul L. Butzer.