Laboratory 3: Variational Calculus. Extremals

Find the extremals for the following integral functionals:

1. Euler-Lagrange Equation:

   (a) $I[y] = \int_0^1 (y'(x)^2 + y'(x) + 1)dx,\ y(0) = 1,\ y(1) = 2$

   (b) $I[y] = \int_1^e (xy'(x)^2 - 2y'(x))dx,\ y(1) = 1,\ y(e) = 2$

   (c) $I[y] = \int_1^2 (y'(x)^2 - xy'(x) - y(x))dx,\ y(1) = 3,\ y(2) = 4$

   (d) $I[y] = \int_1^2 \frac{y'(x)^2}{x}dx,\ y(1) = 1,\ y(2) = 2.$

2. Euler-Lagrange System:

   (a) $\begin{cases} I[y,z] = \int_0^{\pi/2}(y'(x)^2 + z'(x)^2 + 2y(x)z(x))dx, \\ y(0) = z(0) = 0,\ y(\pi/2) = 1,\ z(\pi/2) = -1 \end{cases}$

   (b) $\begin{cases} I[y,z] = \int_2^3(xy'(x) + z'(x)^2 + xy'(x)z'(x))dx \\ y(2) = \ln 3,\ y(3) = \ln 3,\ z(2) = -\ln 2,\ z(3) = 0 \end{cases}$

   (c) $\begin{cases} I[y,z] = \int_0^{\pi/2}(2y(x)z(x) - 2y(x)^2 + y'(x)^2 - z'(x)^2)dx \\ y(0) = 1,\ y(\pi/2) = 1,\ z(0) = 1,\ z(\pi/2) = 1. \end{cases}$

3. Euler-Poisson Equation:

   (a) $\begin{cases} I[y] = \int_0^1(y''(x)^2 - 2y''(x)e^x)dx, \\ y(0) = 0,\ y'(0) = 0,\ y(1) = 2,\ y'(1) = 1 \end{cases}$

   (b) $\begin{cases} I[y] = \int_0^{\frac{\pi\sqrt{2}}{2}}(2y(x)y'(x) - 2y(x)^2 - y'(x)^2 + y''(x)^2)dx, \\ y(0) = 0,\ y'(0) = 0,\ y(\frac{\pi\sqrt{2}}{2}) = 0,\ y'(\frac{\pi\sqrt{2}}{2}) = 0 \end{cases}$

   (c) $\begin{cases} I[y] = \int_0^1(4y''(x)^2 + y'(x)^2 + x^2y(x))dx, \\ y(0) = 1,\ y'(0) = -3,\ y(1) = \frac{1}{24},\ y'(1) = \frac{7}{6} \end{cases}$