Laboratory 8: Mathematical models in population dynamics

1. Let's consider the Gompertz model used in the growth of tumors

$$\begin{cases} x'(t) &= r \cdot x(t) \cdot \ln\left(\frac{K}{x(t)}\right) \\ x(0) &= x_0 \end{cases}$$

where $x_0 > 0$ and $r > 0$.

   (a) Find the model solution;

   (b) Make numerical simulation.

2. Let's consider the mathematical model used in the growth of cells

$$\begin{cases} x'(t) &= \frac{bx(t)}{1+x(t)} - dx(t) \\ x(0) &= x_0 \end{cases}$$

where b is the cells birth rate, d is the cells death rate. The factor $\frac{1}{1+x(t)}$ simulates the crowding effect.

   (a) Show that the equation can be written in the form

$$x'(t) = \frac{r \cdot x(t)}{1 + x(t)} \cdot \left(1 - \frac{x(t)}{K}\right),$$

   where $r = b - d$, $K = \frac{b}{d} - 1$.

   (b) Make numerical simulation.

3. Let consider the growth model with the threshold for a population harvested with the constant rate

$$\begin{cases} x'(t) &= r \cdot x(t) \left[\frac{x(t)}{T} - 1\right] - N \\ x(0) &= x_0 \end{cases}$$

   (a) Try to find the general solution;

   (b) Draw the solution in the case of $r = 1$, $T = 10$, $N = 5$ and for different values of initial population $x_0 = 3$, $x_0 = 5$, $x_0 = 7$, $x_0 = 10$, $x_0 = 12$, $x_0 = 15$, $x_0 = 30$. What is the conclusion?