Laboratory 3: Solving Difference Equations with MAPLE

1. Find the solution for the following difference equations:

   (a) $y(n+1) - 2y(n) = 0$, $y(0) = 3$;
   (b) $y(n+1) - 2y(n) = 4 \cdot 3^n$, $y(0) = 2$;
   (c) $y(n+2) - 3y(n+1) + 2y(n) = 2n^2 + 6n$, $y(0) = 1$, $y(1) = 2$;
   (d) $y(n+2) - 3y(n+1) + 2y(n) = 3^n(2n^2 + 4n)$, $y(0) = 2$, $y(1) = 1$;

2. Consider the simple interest formula $S_n = (1 + np)S_0$ and the compound interest formula $S_n = (1 + p/r)^n S_0$. There are three options to earn interest. Company A offers simple interest at a rate of 6%. Company B offers compound interest at a 4% rate with a conversion period of one month. Company C offers compound interest at a 4% rate with a conversion period of three months.

   (a) Calculate for the three cases the amount on deposit after 5, 10, 15, and 20 years for any principal S_0.
   (b) Which interest offer maximizes the amount on deposit after 5, 10, 15,and 20 years?

3. The loan on a house is $200,000.

   (a) Calculate the monthly repayment needed to have the loan repaid after 30 years. The interest rate is 5%.
   (b) Calculate the total amount paid back on the loan.

4. Let's consider the National Income Model

$$y_{n+2} = \alpha(1 + \beta)y_{n+1} - \alpha\beta y_n + \gamma.$$

   Find the solution in the following cases:

   (a) $\alpha = 0.5$, $\beta = 0$, $\gamma = 1$
   (b) $\alpha = 0.5$, $\beta = 2$, $\gamma = 1$
   (c) $\alpha = 0.6$, $\beta = 2$, $\gamma = 1$
   (d) $\alpha = 0.8$, $\beta = 4$, $\gamma = 1$

   using the initial conditions $y(0) = 0$, $y(1) = 1$. Plot the solutions.

5. Consider the following difference equation:

$$x_{n+1} = \frac{a \cdot x_n + b}{c \cdot x_n + d}$$

   such that $c \neq 0$, $ad - bc \neq 0$.

   (a) Making the substitution $c \cdot x_n + d = \frac{y_{n+1}}{y_n}$ you will get a second order linear difference equation
   (b) Solve the equation

$$x_{n+1} = \frac{2 \cdot x_n + 3}{3 \cdot x_n + 2}$$