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ON MULTIFUNCTION SPACE $\theta L(X)$

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Abstract. In 1982, Christensen [1] studied upper semicontinuous functions and compact valued set-valued mappings. Following that we have introduced the notion of θ-upper (θ-lower) semicontinuous functions. In this paper our main interest of study is $\theta L(X)$, the collection of all θ-cusco maps from a Urysohn, H-closed space X to the space $\mathbb{R}$ of real numbers. We first define the multifunction space $\theta L(X)$ and then prove an important embedding theorem.

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Key words. θ-cusco map, θ-locally bounded, θ-upper semicontinuous functions, θ-lower semicontinuous functions.

1. INTRODUCTION

Historically, there have been two hyperspace topologies of particular importance: the Vietoris topology and the Hausdorff metric topology, as considered by Michael [6] in his fundamental article on hyperspaces. Hausdorff [5] first defined a metric on the collection of all nonempty closed subsets of X, where X is a bounded metric space. Another very important and classical hyperspace topology is the Fell topology introduced by J. M. G. Fell [3]. After that, much of the work has been done on hyperspace topology. In [4] the authors have introduced a new hyperspace topology on the collection of all θ-closed subsets of X.

In this paper our main interest of study is $\theta L(X)$, the collection of all θ-cusco maps from a Urysohn, H-closed space X to the space $\mathbb{R}$ of real numbers. $\theta L(X)$ can be considered as a subset of $\theta(X \times \mathbb{R})$ of all nonempty θ-closed subsets of $X \times \mathbb{R}$, by identifying each θ-cusco map with its graph. So $\theta L(X)$ can inherit the hyperspace topologies from $\theta(X \times \mathbb{R})$. Here we first define the multifunction space $\theta L(X)$ and investigate its relationship with the real-valued θ-semicontinuous functions. We introduce some hyperspace topologies and then prove an important embedding theorem that shows that $\theta(X)$ can be considered as a subspace of $\theta L(X)$ with these hyperspace topologies.

2. THE SPACE $\theta L(X)$ AND θ-SEMICONTINUOUS FUNCTIONS

In this section we study the basic notions of the space $\theta L(X)$ of multifunctions on a topological space X. We then examine the relationship between the space $\theta L(X)$ and the real-valued θ-semicontinuous functions.

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Definition 1. Let X and Y be nonempty sets. A *set-valued mapping* or *multifunction* from X to Y is a mapping that assigns to each element of X, a (possibly empty) subset of Y. If T is a set-valued mapping from X to Y, then its graph is $G(T) = \{(x, y) \in X \times Y : y \in T(x)\}$.

Also, if F is a subset of $X \times Y$ and $x \in X$, then define $F(x) = \{y \in Y : (x, y) \in F\}$. To each subset F of $X \times Y$, a set-valued mapping from X to Y is defined which assigns $F(x)$ to each point $x \in X$. Then F is the graph of a set-valued mapping. Thus, each subset of $X \times Y$ is viewed as a multifunction and every multifunction is viewed as a subset of $X \times Y$ by identifying it with its graph.

Definition 2. Let X and Y be two topological spaces and let T be a set-valued mapping from X to Y. Then T is said to be *θ-upper semicontinuous* (*θ-usc*) at $x \in X$, if whenever V is an open subset of Y containing $T(x)$, then V contains $T(z)$ for each $z \in \operatorname{cl} U$, where U is a neighbourhood of x. T is said to be *θ-upper semicontinuous on* X if it is θ-usc at each point $x \in X$.

Definition 3 ([8]). A T_2 space X is called *H-closed* if any open cover of X by means of open sets in X has a finite proximate subcover i.e., a finite collection whose union is dense in X.

A set $A \subseteq X$ is called an *H-set* if any open cover $\{U_\alpha : \alpha \in \Lambda\}$ of A by open sets of X has a finite subfamily $\{U_{\alpha_i} : i = 1, 2, ..., n\}$ such that $A \subseteq \bigcup_{i=1}^{n} \operatorname{cl} U_{\alpha_i}$.

Definition 4. A multifunction T from X to Y is said to be *θ-usco on* X if T is a θ-usc map such that $T(x)$ is a nonempty H-set in Y for each $x \in X$.

T is said to be *θ-cusc on* X if T is a θ-usc map such that $T(x)$ is a nonempty θ-connected subset of Y for each $x \in X$ (a subset A of X is called *θ-connected* [7] if it is connected in $(X, \operatorname{cl}_\theta)$).

T is said to be *θ-cusco on* X if T is both θ-cusc and θ-usco.

The family of all θ-cusco maps from a Urysohn H-closed space X to the space $\mathbb{R}$ of real numbers is denoted by $\theta L(X)$.

Definition 5. Let X be a Urysohn space. A subset F of $X \times \mathbb{R}$ is said to be *θ-locally bounded at* $x \in X$ if there exist some positive $b \in \mathbb{R}$ and a neighbourhood U of x such that $F(x) \subseteq [-b, b]$, for all $z \in \operatorname{cl} U$. F is said to be *θ-locally bounded on* X if it is θ-locally bounded at each $x \in X$.

Definition 6 ([8]). A point $x \in X$ is said to be a *θ-contact point* of a set $A \subseteq X$ if for every neighbourhood U of x, we get $\operatorname{cl} U \cap A \neq \emptyset$.

The set of all θ-contact points of a set A is called the *θ-closure of* A, and we denote this set by $\operatorname{cl}_\theta A$. A set $A \subseteq X$ is called *θ-closed* if $A = \operatorname{cl}_\theta A$. A set A is called *θ-open* if $X \setminus A$ is θ-closed.

The collection of all θ-open sets in X forms a topology τ_θ on X which is coarser than the original topology of X. We shall denote $\theta(X) = \{A \subseteq X : A$ is nonempty θ-closed$\}$.

THEOREM 1. [2] *In an H-closed Urysohn space, every H-set is θ-closed and every θ-closed set is an H-set.*

PROPOSITION 1. *Let X be a Urysohn, H-closed space. A subset F of $X \times \mathbb{R}$ is the graph of a θ-usco map if and only if F is θ-closed and θ-locally bounded with $F(x)$ nonempty for each $x \in X$.*

Proof. First suppose that F is the graph of a θ-usco map. Let $(x, y) \in \mathrm{cl}_\theta F$. If possible, let$(x, y) \notin F$ i.e., $y \notin F(x)$. Since X is H-closed, Urysohn and $F(x)$ is an H-set, it is θ-closed. Thus there exist some open set V containing $F(x)$ and W containing y such that $V \cap \mathrm{cl}\, W = \emptyset$. Since F is θ-usco, there exists a neighbourhood U of x such that $F(z) \subseteq V$, for all $z \in \mathrm{cl}\, U$. Thus $(\mathrm{cl}\, U \times \mathrm{cl}\, W) \cap F = \emptyset \Rightarrow \mathrm{cl}\,(U \times W) \cap F = \emptyset$ which contradicts the fact that $(x, y) \in \mathrm{cl}_\theta F$. Hence $(x, y) \in F$ and so F is θ-closed. Also, since for each $x \in X$, $F(x)$ is an H-set of $\mathbb{R}$ and F is θ-usco, F is θ-locally bounded on X.

Conversely, suppose that F is a θ-closed, θ-locally bounded subset of $X \times \mathbb{R}$ with $F(x)$ nonempty for each $x \in X$. We have to prove that F is the graph of a θ-usco map at each $x \in X$. If not, then F is not the graph of a θ-usco map at some $x \in X$. Since F is θ-locally bounded at x, there exist some positive $b \in \mathbb{R}$ and some neighbourhood U_x of x such that $F(z) \subseteq [-b, b]$, for all $z \in \mathrm{cl}\, U_x$. Also, since F is not θ-usco at x, there exists some open set V of $\mathbb{R}$ such that $F(x) \subseteq V \subseteq [-b, b]$ and for every neighbourhood U of x contained in U_x, there exists some $x_U \in \mathrm{cl}\, U$ with $y_U \in F(x_U) \setminus V$. Then the net $\{y_U\}$ is contained in $[-b, b] \setminus V$ and so has a θ-cluster point y in $[-b, b] \setminus V$. Hence $\{(x_U, y_U)\}$ is a net in F having a θ-cluster point (x, y) with $(x, y) \notin F$. This contradicts the fact that F is θ-closed and so F is the graph of a θ-usco map. Since for each $x \in X$, $F(x)$ is θ-closed, $F(x)$ is an H-set (since $\mathbb{R}$ is Urysohn, every θ-closed set is an H-set). Hence F is the graph of a θ-usco map. □

COROLLARY 1. *Let X be a Urysohn, H-closed space. The set $\theta L(X)$ is the same as the set of all θ-closed, θ-locally bounded subsets A of $X \times \mathbb{R}$ such that $A(x)$ is an interval in $\mathbb{R}$.*

We next study some basic properties of real-valued θ-semicontinuous functions and investigate their relationship with the space $\theta L(X)$.

DEFINITION 7. A real-valued function defined on a topological space X is called *θ-lower* (respectively, *θ-upper*) *semicontinuous* if for every $x \in X$ and every real number r satisfying the inequality $f(x) > r$ (respectively, $f(x) < r$), there exists a neighbourhood U of x in X such that $f(z) > r$ (respectively, $f(z) < r$), for all $z \in \mathrm{cl}\, U$.

DEFINITION 8. A topological space X is said to be *countably H-closed* if for any countable θ-open cover $\{U_n : n \in \mathbb{N}\}$ of X, there exists a finite subcollection $\{U_i : i = 1, 2, ..., p\}$ such that $X = \mathrm{cl}\,(\bigcup_{i=1}^{p} U_i)$.

Proposition 2. *A topological space X is countably H-closed if and only if every θ-lower (respectively, θ-upper) semicontinuous function on X is bounded below (respectively, bounded above).*

Proof. We prove the result for θ-lower semicontinuous functions. The case of θ-upper semicontinuous functions can be done similarly. First let X be countably H-closed and f be a θ-lower semicontinuous function. Now, by θ-lower semicontinuity of f, $\mathcal{U} = \{f^{-1}(-n, \infty) : n \in \mathbb{N}\}$ is a countable θ-open cover of X and since X is countably H-closed, there exists $m \in \mathbb{N}$ such that $\{f^{-1}[-i, \infty) : i = 1, 2, ..., m\}$ covers X. Thus, for each $x \in X$, $f(x) \geq -m$ and hence f is bounded below.

Conversely, let every θ-lower semicontinuous function on X be bounded below. We now prove that X is countably H-closed. Let $\{U_n : n \in \mathbb{N}\}$ be a countable θ-open cover of X. Without loss of generality, let us assume that $U_n \subseteq U_{n+1}$ for each $n \in \mathbb{N}$. Let $U_0 = \emptyset$. Define a function $f : X \to \mathbb{R}$ by $f(x) = -n$ if $x \in \operatorname{cl} U_n \setminus \operatorname{cl} U_{n-1}$. Then f is clearly a θ-lower semicontinuous function, and hence it is bounded below. Therefore there exists $m \in \mathbb{N}$ such that for each $n \geq m$, $\operatorname{cl} U_n = \operatorname{cl} U_m = X$. Hence X is countably H-closed. □

Definition 9. Let $A \in \theta L(X)$. The real-valued functions a_1 and a_2 on X are said to be the *θ-lower* and *θ-upper boundaries* for A respectively, if for each $x \in X$, $a_1(x) = min\{t : t \in A(x)\}$ and $a_2(x) = max\{t : t \in A(x)\}$.

Lemma 1. *The real-valued functions a_1 and a_2 defined on X are the θ-lower and θ-upper boundaries, respectively, for an $A \in \theta L(X)$ if and only if $a_1 \leq a_2$ and a_1 and a_2 are θ-lower and θ-upper semicontinuous, respectively.*

Proof. Let a_1 and a_2 be the θ-lower and θ-upper boundaries for an $A \in \theta L(X)$. Let $x \in X$. We shall show that a_2 is θ-upper semicontinuous at x. The argument that a_1 is θ-lower semicontinuous at x is similar. Since $A \in \theta L(X)$ is θ-locally bounded at x, there exist a neighbourhood U' of x and a positive $b \in \mathbb{R}$ such that for every $x' \in \operatorname{cl} U'$, $A(x') \subseteq [-b, b]$. If possible, let a_2 be not θ-upper semicontinuous at x. Then there exists $\epsilon > 0$ such that for every neighbourhood U of x contained in U', there exists some $x_U \in \operatorname{cl} U$ with $a_2(x_U) \geq a_2(x) + \epsilon$. Then the net $\{(a_2(x_U)\}$ is contained in $[a_2(x) + \epsilon, b]$ and so it has a θ-cluster point $t \geq a_2(x) + \epsilon$. Then (x, t) is a θ-accumulation point of A, so that $t \in A(x)$ i.e., $t \leq a_2(x)$, which is a contradiction. Hence a_2 is θ-upper semicontinuous at x.

Conversely, let a_1 and a_2 be respectively θ-lower and θ-upper semicontinuous functions such that $a_1 \leq a_2$. Define $A = \{(x, t) \in X \times \mathbb{R} : a_1(x) \leq t \leq a_2(x)\}$. We shall first show that A is θ-locally bounded. Let $x \in X$. Then by the definitions of θ-lower and θ-upper semicontinuity, there exists a neighbourhood U of x such that for every $x' \in \operatorname{cl} U$, $a_1(x) - 1 < a_1(x') \leq a_2(x') < a_2(x) + 1$. Hence A is θ-locally bounded at x. Next we show that A is θ-closed. Let $\{(x_i, y_i)\}$ be a net in A θ-converging to (x, y) in $X \times \mathbb{R}$. If $(x, y) \notin A$, then either $y < a_1(x)$ or $y > a_2(x)$, say the latter. Let $s \in \mathbb{R}$ be such that $a_2(x) < s < y$.

Then x has a neighbourhood U such that for every $x' \in \operatorname{cl} U$, $a_2(x') < s$. But as $\{x_i\}$ is cofinally in $\operatorname{cl} U$, $y \leq s$, a contradiction. Therefore A is θ-closed and so $A \in \theta L(X)$ having a_1 and a_2 as its θ-lower and θ-upper boundaries. □

DEFINITION 10. By $M(X)$, we denote the set of all pairs (f, g) where f, g are real-valued functions defined on X such that for each $x \in X$, $f(x) < g(x)$. For $(f, g) \in M(X)$, we define the set

$$W(f, g) = \{(x, t) \in X \times \mathbb{R} : f(x) < t < g(x)\}.$$

LEMMA 2. *Let $(f, g) \in M(X)$. Then f is θ-upper semicontinuous and g is θ-lower semicontinuous if and only if $W(f, g)$ is a θ-open subset of $X \times \mathbb{R}$.*

Proof. Similar to that of Lemma 1. □

PROPOSITION 3. *For each real-valued continuous function f defined on X, and a θ-open set W of $X \times \mathbb{R}$ containing f, there exist a θ-upper semicontinuous function g and a θ-lower semicontinuous function h on X such that $f \subseteq W(g, h) \subseteq W$.*

Proof. For each $x \in X$, since $(x, f(x)) \in W$, we can find an open subset U_x of X and a positive $r_x < 1$ such that $(x, f(x)) \in \operatorname{cl} U_x \times [f(x) - r_x, f(x) + r_x] \subseteq W$ and $f(\operatorname{cl} U_x) \subseteq [f(x) - r_x, f(x) + r_x]$. Define $W_0 = \bigcup\{\operatorname{cl} U_x \times [f(x) - r_x, f(x) + r_x] : x \in X\}$. Then W_0 is a θ-open subset of $X \times \mathbb{R}$ such that for each $x \in X$, $W_0(x)$ is an interval in $\mathbb{R}$. Also, for every $x \in X$, $W_0(x) = \cup\{[f(z) - r_z, f(z) + r_z] : z \in X \text{ and } x \in \operatorname{cl} U_z\} \subseteq [f(x) - 2, f(x) + 2]$, which shows that $W_0(x)$ is bounded for each $x \in X$. Let g and h denote respectively the θ-lower and θ-upper boundaries of W_0 i.e., for each $x \in X$, $g(x) = \inf W_0(x)$ and $h(x) = \sup W_0(x)$. Then $W_0 = W(g, h)$ and so by Lemma 2, g is θ-upper semicontinuous and h is θ-lower semicontinuous on X. □

3. EMBEDDING THEOREM IN HYPERSPACE TOPOLOGY

In this section we first introduce new hyperspace topologies on the collection $\theta(X)$ of all nonempty θ-closed subsets of X. We then give a very important embedding theorem.

DEFINITION 11. Let (X, τ) be a topological space. For $U \subseteq X$, define $U^+ = \{A \in \theta(X) : A \subseteq U\}$ and $U^- = \{A \in \theta(X) : A \cap U \neq \emptyset\}$. Then:

(i) The sets of the form $V_1^- \cap V_2^- \cap ... \cap V_n^- \cap V_0^+$ where V_1, V_2,...,V_n are open sets and V_0 is a θ-open set, is a base for some topology τ_V on $\theta(X)$.

(ii) The sets of the form $V_1^- \cap V_2^- \cap ... \cap V_n^- \cap V_0^+$ where V_1, V_2,...,V_n are open sets and V_0 is a θ-open set with $X \setminus V_0$ an H-set, is a base for some topology $\tau_F$on $\theta(X)$ [4].

(iii) The topology τ_{V^-} on $\theta(X)$ is generated by a subbase consisting of all sets of the form G^- where G is open in X. Similarly, the topology τ_{V^+} (respectively, co-H-set topology τ_H) is generated by all sets of the form V^+, where V is θ-open in X (respectively, whose complement is an H-set in X).

The supremum $\tau_{V^-} \vee \tau_{V^+}$ (respectively, $\tau_{V^-} \vee \tau_H$) is the topology τ_V (respectively, τ_F) on $\theta(X)$.

Note that since $\theta L(X) \subseteq \theta(X \times \mathbb{R})$, $\theta L(X)$ can inherit each of the aforementioned hyperspace topologies from $\theta(X \times \mathbb{R})$.

THEOREM 2. *The following statements hold:*

(i) *The space $\theta_{\tau_{V^+}}(X)$ is embeddable in $\theta L_{\tau_{V^+}}(X)$.*
(ii) *The space $\theta_{\tau_{V^-}}(X)$ is embeddable in $\theta L_{\tau_{V^-}}(X)$.*
(iii) *The space $\theta_{\tau_V}(X)$ is embeddable in $\theta L_{\tau_V}(X)$.*
(iv) *The space $(\theta(X) \cup \{\emptyset\})_{\tau_H}$ is embeddable in $\theta L_{\tau_H}(X)$.*
(v) *The space $\theta_{\tau_H}(X)$ is embeddable in $\theta L_{\tau_H}(X)$.*
(vi) *The space $\theta_{\tau_F}(X)$ is embeddable in $\theta L_{\tau_F}(X)$.*
(vii) *The space $(\theta(X) \cup \{\emptyset\})_{\tau_F}$ is embeddable in $\theta L_{\tau_F}(X)$.*

Proof. For each $E \in \theta(X) \cup \{\emptyset\}$, define

$$F_E = (X \times \{0\}) \cup (E \times [0,1])$$

and the sets $\mathcal{F} = \{F_E : E \in \theta(X)\}$ and $\mathcal{F}_\emptyset = \{F_E : E \in \theta(X) \cup \{\emptyset\}\}$. Then $\mathcal{F}$ and $\mathcal{F}_\emptyset$ are contained in $\theta L(X)$. Define $\Phi : \theta(X) \cup \{\emptyset\} \to \theta L(X)$ by $\Phi(E) = F_E$ for each $E \in \theta(X) \cup \{\emptyset\}$ and denote the restriction of Φ to $\theta(X)$ by Φ_0. Then Φ and Φ_0 are one-to-one.

(i) We prove that Φ_0 is a homeomorphism from $\theta_{\tau_{V^+}}(X)$ to $\theta L_{\tau_{V^+}}(X)$. Let $A \in \theta(X)$ and let W^+ be an open neighbourhood of F_A in $\theta L_{\tau_{V^+}}(X)$, where W is θ-open in $X \times \mathbb{R}$. Since $[0,1]$ is an H-set, there exists an open subset U of X such that $A \subseteq U$ and $U \times [0,1] \subseteq W$. Now let $B \in U^+ \cap \theta(X)$. Then $\Phi_0(B) = F_B \in W^+$. Hence Φ_0 is continuous on $\theta_{\tau_{V^+}}(X)$. Next let $A \in \theta(X)$ and U be a θ-open subset of X such that $A \in U^+$. Then $W = (X \times (-\frac{1}{2}, \frac{1}{2})) \cup (U \times \mathbb{R})$ is a θ-open set in $\mathbb{R}$ such that $F_A \in W^+$ and $W^+ \cap \Phi_0(\theta(X)) \subseteq \Phi_0(U^+)$. Hence Φ_0 is a homeomorphism from $\theta_{\tau_{V^+}}(X)$ to $\theta L_{\tau_{V^+}}(X)$.

(ii) We show that Φ_0 is a homeomorphism from $\theta_{\tau_{V^-}}(X)$ to $\theta L_{\tau_{V^-}}(X)$. Let $A \in \theta(X)$ and W^- be an open neighbourhood of F_A in $\theta L_{\tau_{V^-}}(X)$, where W is open in $X \times \mathbb{R}$. Let $(x,t) \in W \cap F_A$. If $t = 0$, then $\Phi_0(\theta(X)) \subseteq W^-$. So let $t \neq 0$ and choose an open neighbourhood U of x and an open interval V containing t such that $(x,t) \in U \times V \subseteq W$. Then if $B \in U^- \cap \theta(X)$, then $F_B \in W^-$. Similarly, if U^- is an open neighbourhood of $A \in \theta(X)$, then $(U \times (\frac{1}{2}, 2))^- \cap \Phi_0(\theta(X)) \subseteq \Phi_0(U^-)$. Hence Φ_0 is a homeomorphism from $\theta_{\tau_{V^-}}(X)$ to $\theta L_{\tau_{V^-}}(X)$.

(iii) It follows from (i) and (ii).

(iv) We show that Φ is a homeomorphism from $(\theta(X) \cup \{\emptyset\})_{\tau_H}$ to $\theta L_{\tau_H}(X)$. Let $E \in \theta(X) \cup \{\emptyset\}$ and let K be an H-set of $X \times \mathbb{R}$ such that $F_E \cap K = \emptyset$. Without loss of generality, let $K \subseteq X \times [0,1]$. Then $X(K) = \{x \in X : (x,t) \in K \text{ for } t \in [0,1]\}$ is an H-set.

Let $A \in (X \setminus X(K))^+$. Then by definition of $X(K)$, for any $x \in A$ and $t \in [0,1]$, $(x,t) \notin K$. Again since $F_E \cap K = \emptyset$, $(X \times \{0\}) \cap K = \emptyset$. Hence $F_A \in (K^c)^+$ (where K^c denotes the complement of K) and thus Φ is continuous on $(\theta(X) \cup \{\emptyset\})_{\tau_H}$. To show that Φ is open, let K_0 be an H-set in X and let $E \in (X \setminus K_0)^+ \cap \theta(X)$. Let $K = K_0 \times \{1\}$. Then $F_E \in (K^c)^+ \cap \mathcal{F}_\emptyset \subseteq \Phi((X \setminus K_0)^+)$. Hence Φ is a homeomorphism from $(\theta(X) \cup \{\emptyset\})_{\tau_H}$ to $\theta L_{\tau_H}(X)$.

(v) It follows from (iv) above.

(vi) It follows from (ii) and (v).

(vii) We prove that Φ is a homeomorphism from $(\theta(X) \cup \{\emptyset\})_{\tau_F}$ to $\theta L_{\tau_F}(X)$. Note that for $\emptyset$, any basic open neighbourhood $G^+ \cap \mathcal{G}^- \cap \mathcal{F}_\emptyset = G^+ \cap \mathcal{F}_\emptyset$, where G is a subset of $X \times \mathbb{R}$ with G^c an H-set and $F_\emptyset \subseteq G$ and $\mathcal{G}$ is a finite family of open subsets of $X \times \mathbb{R}$ such that $F_\emptyset \in \mathcal{G}^-$. Then arguing in the same way as in (iv), Φ becomes continuous. Also, by (ii) and (iv), Φ is continuous at each $E \in \theta(X)$. In a similar way, Φ is an open map from $(\theta(X) \cup \{\emptyset\})_{\tau_F}$ to $\theta L_{\tau_F}(X)$. □

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