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VIETORIS TOPOLOGY ON HYPERSPACES ASSOCIATED TO A NONCOMMUTATIVE COMPACT SPACE

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Abstract. We study some topological spaces that can be considered as hyperspaces associated to noncommutative spaces. More precisely, for a NC compact space associated to a unital C^*-algebra, we consider the set of closed projections of the second dual of the C^*-algebra as the hyperspace of closed subsets of the NC space. We endow this hyperspace with an analog of Vietoris topology. In the case that the NC space has a quantum metric space structure in the sense of Rieffel we study the analogs of Hausdorff and infimum distances on the hyperspace. We also formulate some problems about distances between sub-circles of a quantum torus.

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Key words. C^*-algebra, state space, closed projection, hyperspace, Vietoris topology, Hausdorff distance, infimum distance.

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