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SOME PROPERTIES OF SOLUTIONS OF THE HOMOGENEOUS NONLINEAR SECOND ORDER DIFFERENTIAL EQUATIONS

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Abstract. In this paper we consider the following nonlinear homogeneous second order differential equations, $F(x, y, y', y'') = 0$. We present for the solutions, $y \in C^2[a, b]$, of this equation, extremal principle, Sturm-type, Nicolescu-type and Butlewski-type separation theorems. Some applications and examples are given. Open problems are also presented.

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Key words. Homogeneous nonlinear second order differential equation, zeros of solutions, Sturm-type theorem, Nicolescu-type theorem, Butlewski-type theorem, bilocal problem, Cauchy problem, open problem, extremal principle.

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