SOME FIXED POINT THEOREMS
IN TERMS OF TWO MEASURES OF NONCOMPACTNESS

RADU PRECUP and IOAN A. RUS

Abstract. In this paper several fixed point theorems of Sadovskii type are obtained for operators on spaces endowed with two norms and two corresponding measures of noncompactness. An application to Hammerstein integral equations in a Banach space is included to illustrate the theory.

MSC 2010. 47H10, 54H25, 47H08, 47N20

Key words. Linear space with two norms, measure of noncompactness, condensing operator, fixed point, radial retraction, retractible operator, integral equation.

REFERENCES

[1] AKHMEROV, R.R., KAMENSKII, M.I., POTAPOV, A.S., RODKINA, A.E. and SADOVSKII, B.N., Measures of Noncompactness and Condensing Operators, Birkhäuser, Basel, 1992.

[2] AMBROSETTI, A., *Un teorema di esistenza per le equazioni differenziali negli spazi di Banach*, Rend. Sem. Mat. Univ. Padova, **39** (1967), 349–360.

[3] APPELL, J., *Implicit functions, nonlinear integral equations and the measure of noncompactness of the superposition operator*, J. Math. Anal. Appl., **83** (1981), 251–263.

[4] APPELL, J., *Measure of noncompactness, condensing operators and fixed points: an application-oriented survey*, Fixed Point Theory, **6** (2005), 157–229.

[5] BANAS, J. and GOEBEL, K., *Measure of Noncompactness in Banach Spaces*, M. Dekker, New York, 1980.

[6] BROWN, R.F., *Retraction methods in Nielson fixed point theory*, Pacific J. Math., **115** (1984), 277–297.

[7] DEIMLING, K., *Nonlinear Functional Analysis*, Springer, Berlin, 1985.

[8] DE PASCALE, E., TROMBETTA, G. and WEBER, H., *Convexly totally bounded and strongly totally bounded sets. Solution of a problem of Idzik*, Ann. Sc. Norm. Super. Pisa Cl. Sci. (5), **20** (1993), 341–355.

[9] FORT, M.K., *Essential and nonessential fixed points*, Amer. J. Math., **72** (1950), 315–322.

[10] GRANAS, A., *Points fixes pour les applications compactes:espaces de Lefschetz et la théorie de l'indice*, Les Presses de l'Université de Montréal, 1980.

[11] GUO, D., LAKSHMIKANTHAM, V. and LIU, X., *Nonlinear Integral Equations in Abstract Cones*, Kluwer, Dordrecht, 1996.

[12] HEINZ, H.P., *On the behaviour of measures of noncompactness with respect to differentiation and integration of vector-valued functions*, Nonlinear Anal., **7** (1983), 1351–1371.

[13] MEEHAN, M. and O'REGAN, D., *Existence theory for nonlinear Fredholm and Volterra integral equations on half-open intervals*, Nonlinear Anal., **35** (1999), 355–387.

[14] NICULESCU, C.P. and ROVENTA, I., *Schauder fixed point theorem in spaces with global nonpositive curvature*, Fixed Point Theory Appl., 2009, Article ID906727, 8 pages.

[15] O'REGAN, D. and PRECUP, R., *Existence criteria for integral equations in Banach spaces*, J. Inequal. Appl., **6** (2001), 77–97.

[16] O'REGAN, D. and PRECUP, R., *Theorems of Leray-Schauder Type and Applications*, Gordon and Breach, Amsterdam, 2001.

[17] O'REGAN, D. and PRECUP, R., *Existence theory for nonlinear operator equations of Hammerstein type in Banach spaces*, Dynam. Systems Appl., **14** (2005), 121–134.

[18] PETRYSHYN, W.P., *Fixed point theorems for various classes of 1-set-contractive and 1-ball contractive mappings in Banach spaces*, Trans. Amer. Math. Soc., **182** (1973), 323–352.

[19] PRECUP, R., *Methods in Nonlinear Integral Equations*, Kluwer, Dordrecht, 2002.

[20] RUS, I.A., *A general fixed point principle*, Seminar on Fixed Point Theory, Cluj-Napoca, 1985, 69–76.

[21] RUS, I.A., *Fixed Point Structure Theory*, Cluj University Press, 2006.

[22] RUS, I.A., *Five open problems in fixed point theory in terms of fixed point structures (I): singlevalued operators*, Proc. 10th IC-FPTA, Cluj-Napoca, 2013, 39–60.

[23] WILLIAMSON, T.E., *A geometric approach to fixed points of non-self mappings $T : D \to X$*, Contemp. Math., **18** (1983), 247–253.

Babeş-Bolyai University
Department of Mathematics
Str. M. Kogălniceanu 1
400084 Cluj-Napoca, Romania
E-mail: `r.precup@math.ubbcluj.ro`
E-mail: `iarus@math.ubbcluj.ro`